Wave energy absorption amongst arrays of point absorbers with a non-regular geometry

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1. INTRODUCTION

The interaction theory presented by Kagemoto & Yue (1986) and refined by many other scholars is effective in wave effects on multiple surface-piercing bodies. In this work, by virtue of a boundary element method using the hybrid dipole-source formulation (Liu *et al.* 2021), interactions among point absorber arrays are considered. Based on the fundamental theoretical result that the maximum mean power generated by an axisymmetric heaving converter is equivalent to that contained in $\lambda/2\pi$ length of an incident wave crest (Evans 1979; Falnes 1980), the array performance can be numerically predicted. Given that regular shapes such as cylinders are often used for the individual device in an array, a non-regular geometry is preferred. Moreover, the characteristics in the interactions amongst devices in association with different layouts is also of particular interest.

2. THEORY AND METHODOLOGY

2.1. *Wave scattering amongst multiple floating bodies*

Let us consider a train of regular incident waves that propagates to the positive x-direction and transmits with a small amplitude A, a heading angle β measured from the positive x-axis, and a wave number k, in water of a finite depth h. In terms of polar coordinates (r_j, θ_j, z_j), the ambient wave potential incident to body j can be expressed as a summation of partial cylindrical waves incident to the body j:

$$\phi_j^A(r_j,\theta_j,z_j) = -\frac{\mathrm{i}gA}{\omega}\frac{\cosh k(z_j+h)}{\cosh kh}\mathrm{e}^{\mathrm{i}k(x_j\cos\beta+y_j\sin\beta)}\sum_{q=-\infty}^{\infty}J_q(kr_j)\mathrm{e}^{\mathrm{i}q(\pi/2+\theta_j-\beta)}. \tag{1}$$

Eq. (1) can be further written as the following compact form

$$\phi_j^A(x_j,y_j,z_j) = \{a_j^I\}^T\{\psi_j^I\}, \tag{2}$$

in which $\{\psi_j^I\}$ is a scalar vector of the basis function that is also named as the incident partial wave component

$$\{\psi_j^I\}_{lq} = \begin{cases} \frac{\cosh k(z_j+h)}{\cosh kh}J_q(kr_j)\,\mathrm{e}^{\mathrm{i}q\theta_j}, & l=0 \\ \cos k_l(z_j+h)\,I_q(k_l r_j)\,\mathrm{e}^{\mathrm{i}q\theta_j}, & l\geqslant 1 \end{cases}, \tag{3}$$

where J_q is the Bessel function of the first kind of order q, and I_q is the modified Bessel function of the first kind of order q, l represents numbering of the eigen-expansion mode.

In arrays of floating bodies, each individual body not only experiences the ambient incident plane wave, but also outgoing waves that are scattered from all the other neighboring bodies in the arrays. The scattered waves from body i can then be expressed as incident waves to body j:

$$\phi_i^S(r_i,\theta_i,z_i) = \{A_i^S\}^T[T_{ij}]\{\psi_j^I\}, \tag{4}$$

where $\{A_i^S\}$ is a scalar vector of the scattering coefficients, and the T-transfer matrix is derived from Graf's addition theorem:

$$[T_{ij}]_{nn}^{mq} = \begin{cases} H_{m-q}(kL_{ij})\mathrm{e}^{\mathrm{i}\alpha_{ij}(m-q)}, & n=0 \\ K_{m-q}(k_nL_{ij})\mathrm{e}^{\mathrm{i}\alpha_{ij}(m-q)}(-1)^q, & n\geqslant 1 \end{cases}. \tag{5}$$

where H_{m-q} is the zeroth-order Hankel function of the first kind; K_{m-q} is the zeroth-order modified Bessel function of the second kind; L_{ij} is the distance between the centers (origins of the local coordinate systems) of body i and j; α_{ij} is the angle at body i between the positive x-direction and the line joining the center of i to that of j in an anti-clockwise direction. Note that Eq. (5) holds true for any integer m, and any non-negative integer n, when $r_j \leqslant L_{ij}$. On the basis of Eq. (4), the total wave potentials that incident to body j can be expressed as a summation of the ambient incident plane wave and all scattered waves from the other bodies. Using a so-called diffraction transfer matrix $[D_j]$ as defined in Liu *et al.* (2021) based on the hybrid dipole-source formulation, the scattered wave from body j and all the incident waves to the same body can be connected via

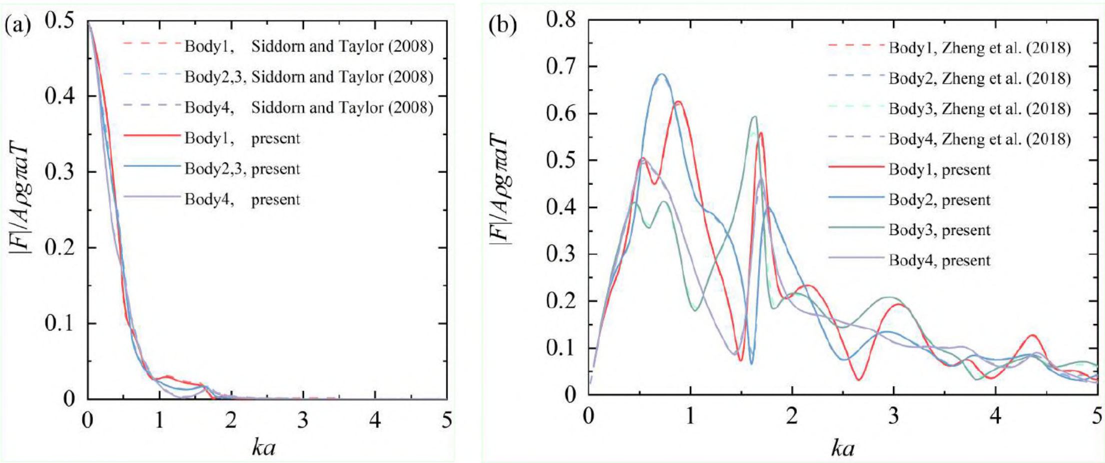

Figure 1: Excitation forces acting on each body of the four-cylinder array in oblique waves, $\beta = \pi/4$: (a) vertical force F_z; (b) horizontal force F_x.

the following relation

$$\{A_j^S\} = [D_j] \cdot (\{a_j^I\} + \sum_{\substack{i=1 \\ i\neq j}}^{N_B} [T_{ij}]^T \{A_i^S\}), \quad (j = 1, 2, ..., N_B). \tag{6}$$

from which the scattering coefficients can be numerically solved, where N_B stands for the number of bodies in the arrays.

Wave excitation forces can then be calculated via pressure integration over each body using matrix operations:

$$F_{j,p}^E = \{\eta_j^E\}\{G_{j,p}^E\}, \tag{7}$$

where $F_{j,p}^E$ is interpreted as the excitation force in the p^{th} degree of freedom of body j. $G_{j,p}^E$ is named as the force transfer matrix with the following form

$$\{G_{j,p}^E\} = \mathrm{i}\omega\rho \iint_{S_B^j} \left(\{\psi_j^I\} + [D_j]^T \{\psi_j^S\}\right) n_{j,p} \mathrm{d}S, \tag{8}$$

where n_p is the p^{th} component of the normal vector on the immersed body surface; S_B^j denotes the immersed body surface; ρ denotes the water density. The overall expansion coefficients of the total waves that incident to body j read

$$\{\eta_j^E\} = \{a_j^I\}^T + \sum_{\substack{i=1 \\ i\neq j}}^{N_B} \{A_i^S\}^T [T_{ij}]. \tag{9}$$

Following a similar approach, wave radiation forces acting on body j can be evaluated as

$$F_{j,t}^{R,i,p} = \begin{cases} \left\{\eta_j^{R,i,p}\right\}\{G_{j,t}^E\}, & i \neq j \\ \mathrm{i}\rho\left(a_{j,p} + \omega b_{j,p}\right) + \left\{\eta_j^{R,i,t}\right\}\{G_{j,t}^E\}, & i = j \end{cases} \tag{10}$$

where $a_{j,p}$ and $b_{j,p}$ are respectively the added mass and the radiation damping of body j in the p^{th} degree of freedom due to its own unitary motion in the same mode when the body is in isolation; $F_{j,t}^{R,i,p}$ is interpreted as the radiation force of body j in the t^{th} degree of freedom due to the motion of body i in p^{th} degree of freedom.

2.2. *Wave power and interaction factor*

It is common to use the so-called interaction factor to evaluate the performance of arrays of wave energy converters. The interaction factor q is defined as a function of the wave number k and the incident wave angle β and can be numerically evaluated by

$$q(k,\beta) = \frac{2\pi}{\lambda N_B} \frac{\left(\sum_{j=1}^{N_B} P_j(k,\beta)\right)_{\max}}{P_W}, \tag{11}$$

where P_j represents the wave power of each converter in the arrays. Physically, $q < 1$ indicates that the wave interactions have a destructive effect on the power absorption of the wave farm. In contrast, if $q > 1$, the park effect is constructive. The maximum absorbed power of an entire array can be calculated as (Evans 1979, etc.)

$$\left(\sum_{j=1}^{N} P_j(k,\beta)\right)_{\max} = \frac{1}{8}\{F^E\}^* [B_{\text{rad}}]^{-1} \{F^E\}, \tag{12}$$

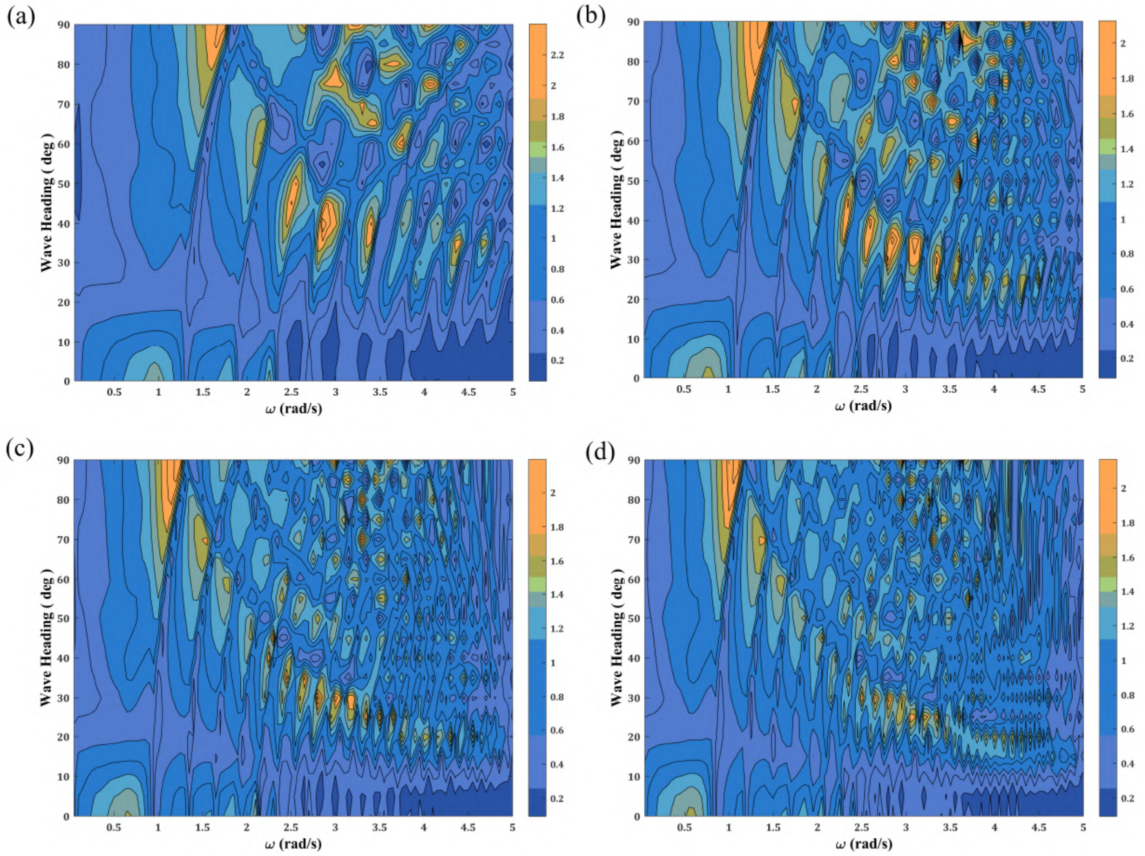

Figure 3: q-factor of a line array of CorPower-like point absorbers against the device spacing s: (a) $s = 2D$; (b) $s = 3D$; (c) $s = 4D$; (d) $s = 5D$ ($h = 50$m).

where $\{F^E\}$ is a scalar vector of complex amplitudes of the wave excitation forces; $[B_{\rm rad}]$ represents the radiation damping matrix; the asterisk $*$ denotes the complex conjugate transpose. In Eq. (11), P_W is the time-averaged incident wave power per unit crest-width and defined by

$$P_W = \frac{1}{4}\rho g A^2 \sqrt{\frac{g}{k}\tanh kh}\left(1 + \frac{2kh}{\sinh 2kh}\right). \tag{13}$$

3. RESULTS AND DISCUSSIONS

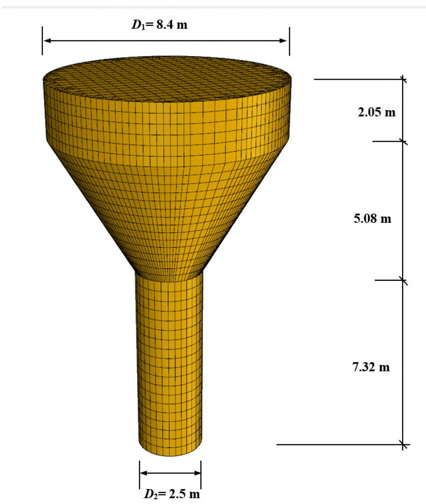

Figure 2: Device geometry and surface meshing.

Verification is firstly performed via a four-cylinder benchmark problem given in Siddorn & Taylor (2008). The geometry of each body is a truncated vertical cylinder of radius $r = a$, draught $T = 2a$, being placed in water of depth $h = 4a$. The foursquare layout of the array separates neighboring cylinders by a distance of $s = 4a$ for an incident wave heading angle of $\beta = \pi/4$. Figure 1 shows that our present results agree well with those from theoretical methods documented in Siddorn & Taylor (2008) and Zheng *et al.* (2018).

Numerical computations are carried out for a line array along the x-axis of 7 evenly-spaced CorPower-like heaving point abosrbers and a double array in parallel involving 7×2 evenly-spaced point abosrbers of the same type. The surface of the individual device is discretised using 3060 panels on the immersed body surface and 184 panels on the waterplane cross-sectional area. The mesh and dimensions of the device are given in Figure 2.

Variations of the q-factor versus the device spacing s, the wave heading angle β, and the wave angular frequency ω are displayed in Figure 3 and 4. It is found that the q-factor distributions of a single array are more regular than those of a double array. There are some remarkable "bright spot" regions indicating that the wave energy absorption there is locally optimised against wave conditions. For a single array, the lower the wave frequency is, the larger the wave heading tends to be in

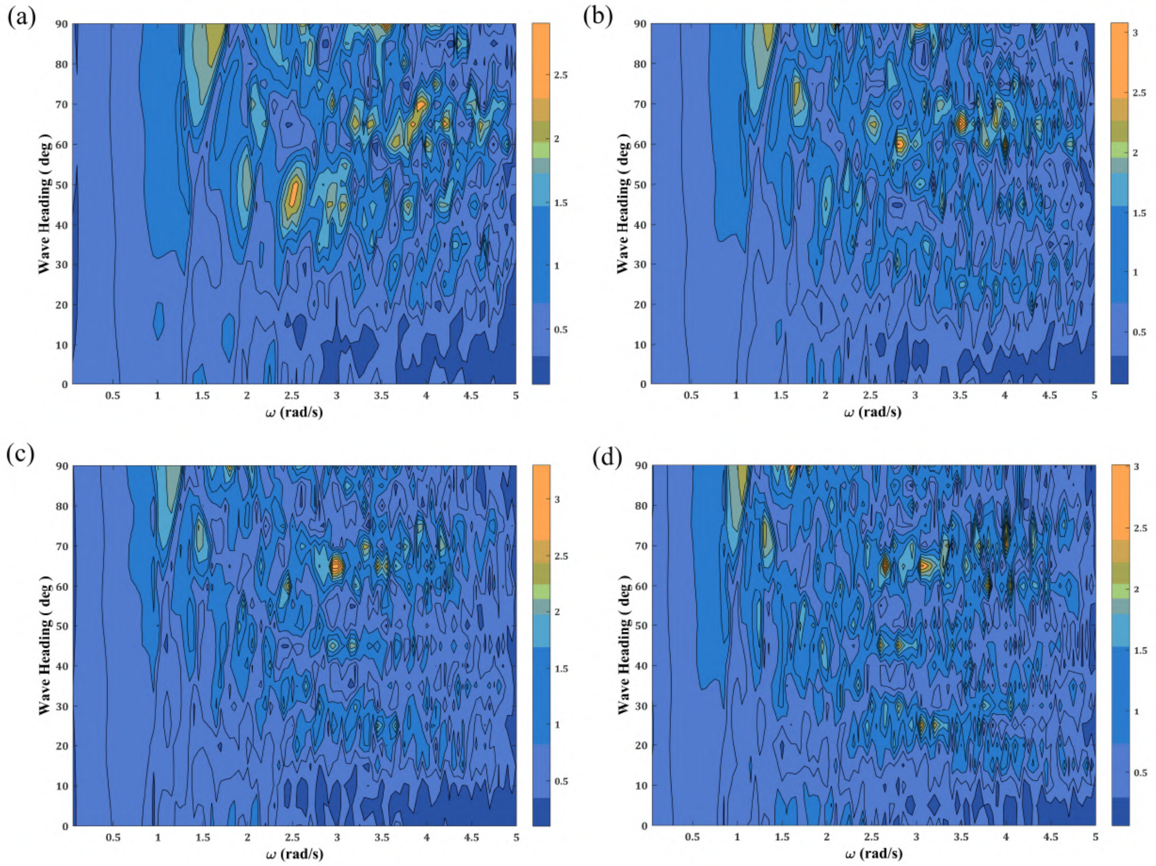

Figure 4: q-factor of a double-parallel array of CorPower-like point absorbers against the device spacing s: (a) $s = 2D$; (b) $s = 3D$; (c) $s = 4D$; (d) $s = 5D$ ($h = 50$m).

association with the centre of a spot region. In contrast, for a double array, the distribution of spot regions are rather randomised, indicating much stronger interactions amongst the devices as the scattered waves not only come from neighboring devices in row but also in column. In addition, both cases suggest that the number (or density) of the spot regions increases with the device spacing. More results will be presented at the workshop regarding other interesting phenomena in the point absorber arrays including those in staggered arrangement.

4. CONCLUSIONS

Wave interactions with multiple axisymmetric point absorbers is presented. Good agreement is found with the theoretical methods for cylinders. The method is applied to non-regularly shaped CorPower-like heaving devices in a line array and a double array in parallel. It is found that there are "bright spot" regions where wave energy absorption is optimised and the distribution of such regions becomes randomised in a double array. Moreover, the density of spot regions increases with the device spacing and the size varies with the frequency.

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