Body-Exact Computations of Vertical Plane Motions

(Abstract for the 37th International Workshop on Water Waves and Floating Bodies, Giardini Naxos, Italy, 2022)

Rahul Subramanian[1], **Robert F. Beck**[2]
[1]Department of Ocean Engineering, Texas A&M University, Galveston, USA
[2]Department of Naval Architecture and Marine Engineering, University of Michigan, Ann Arbor, USA
Presenting author: rbeck@umich.edu

1 INTRODUCTION

Assessing the motion response of a vessel is a crucial component of the ship design process. The heave and pitch motions and subsequent accelerations are critical for assessing the inertial loads at key locations and have bearing on the dynamic structural loading and proper prediction of slamming probabilities and impact.

Computational methods employing the potential flow theory, formulate the problem either in the time-domain or the frequency domain. Although frequency domain codes have proven to be fairly accurate, robust and computationally fast, they are inherently restricted by the assumption of small wave amplitude because of the linear formulation. Time-domain based methods on the other hand have a major advantage in being able to model different degrees of nonlinearity, as reviewed in [1]. The sophistication level of these so called "blended method" codes can be chosen based on the particular problem being solved, the required accuracy level and available computational resources. Validation studies ([2], [3]) have shown increased accuracy in forward speed and capability in handling large motions when compared with linear frequency domain methods.

This paper presents the application of an efficient time-domain "blended-method" formulation to predict the ship motions of a vessel under forward speed. Computations are done specifically for vertical plane motions in head seas in order to compare with experimental data. The computer program is based on the body-exact time-domain strip theory developed by [4], which was extended to predict maneuvering in waves ([5]). Over the years, the physics models have been continuously updated to improve the predictions ([6], [7]). The strip theory approach allows for faster computational times when compared to fully three-dimensional methods. The free surface boundary conditions are transferred to a "representative surface" as explained in the section below. The body boundary condition is applied below the intersection surface of the incident wave and the exact body position at each time step. Accounting for the changing wetted surface ensures that the nonlinear radiation and diffraction wave loads are also captured in addition to the nonlinear Froude-Krylov and hydrostatic loads. Computations have been performed for the KCS and KVLCC2, representing two different modern hull forms. The results have been compared with available experiments and predictions from other numerical methods.

2 MATHEMATICAL FORMULATION

The fluid flow for the wave body problem is considered inviscid, irrotational, incompressible and unsteady. Since most ocean going vessels are relatively slender, a strip theory formulation is adopted, where the three-dimensional problem is solved as a series of individual two-dimensional problems.

The strip wise two-dimensional potential $\phi = \phi(y, z, t; x)$ satisfies the Laplace equation at each frame such that $\nabla^2\phi(y, z, t; x) = 0$. Three different coordinate frames are used as shown in Figure 1. An inertial Earth fixed frame with origin at O_e is used to keep track of the vessel translational motions and the Euler angles. The boundary value problem is formulated in a hydrodynamic frame that translates and rotates in the horizontal plane with the body. A body fixed frame is used to solve the equations of motion. The hydrodynamic frame follows the ship such that its origin O_h is always in a vertical line with the origin of the body fixed frame, O_b.

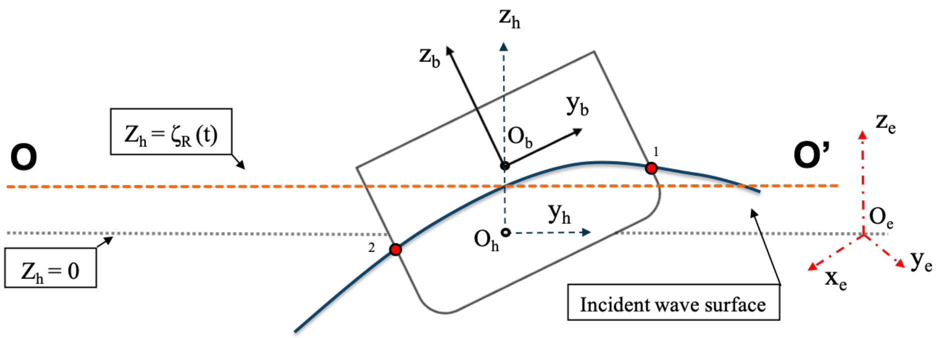

Figure 1: Schematic showing the details at a station. The free surface boundary conditions are transferred to the representative surface given by $z_h = \zeta_R(t)$. The origins of the three frames are denoted by O. (Schematic shows vessel in general non-head sea condition)

In the present formulation, the free surface boundary conditions are applied on a "representative surface", indicated by (line) O-O' in Figure 1 given by $z_h = \zeta_R(t)$. Here, $\zeta_R(t)$ is a representative free-surface at each station, given by $\zeta_R(t) = \eta_I(x_h = x_{station}, y_h = 0, t)$. Here, η_I, $x_{station}$, and y_h represent the incident wave elevation, longitudinal location of a station and the y coordinate y_h in the hydrodynamic frame ($y_h = 0$ indicates intersection of incident wave with z axis of hydrodynamic frame), respectively. The quasi-linear kinematic and dynamic free surface boundary conditions written in the moving hydrodynamic frame, applied on the $z_h = \zeta_R(t)$ plane (line in 2-D) are as follows

$$\frac{\delta \eta}{\delta t} = \frac{\partial \phi}{\partial z} + V\frac{\partial \eta}{\partial y} + x_h\dot{\psi}\frac{\partial \eta}{\partial y} + v_m\frac{\partial \eta}{\partial y} - \mu\eta \tag{1}$$

$$\frac{\delta \phi}{\delta t} = -g\eta + V\frac{\partial \phi}{\partial y} + x_h\dot{\psi}\frac{\partial \phi}{\partial y} + v_m\frac{\partial \phi}{\partial y} + \dot{\zeta}_R(t)\frac{\partial \phi}{\partial z} - \mu\phi \tag{2}$$

Here $\frac{\delta}{\delta t}$ refers to the time derivative taken by following a free surface node. The terms η and x_h represent the free surface wave elevation measured from the calm water surface, and the longitudinal position of the station in the hydrodynamic frame, respectively. Additional terms are picked up to account for the horizontal (v_m) and vertical ($\dot{\zeta}_R(t)$) movement of the free surface nodes wrt the hydrodynamic frame. The moving node velocity v_m accounts for the lateral movement of each free surface node. This is necessary in a body-exact formulation. The last term in the equations (1) and (2) are the damping terms that are added to generate artificial damping at the beach to satisfy the far-field condition at the edge of the domain. Details for the numerical beach can be found in [8].

The body boundary condition can thus be written in terms of its individual components for the diffraction (ϕ_D) and radiation (ϕ_R) potentials,

$$\nabla\phi_D \cdot \mathbf{N} = -\nabla\phi_I \cdot \mathbf{N} \quad \text{on } S_B(t) \tag{3}$$

$$\nabla\phi_R \cdot \mathbf{N} = \tilde{\mathbf{v}} \cdot \mathbf{N} \quad \text{on } S_B(t) \tag{4}$$

The velocity $\tilde{\mathbf{v}}$ used in Equation (4) is the velocity of a point on the body surface after removing the time-varying mean velocity components in the horizontal plane. Therefore, the velocity $\tilde{\boldsymbol{v}}$ is evaluated as $\tilde{\mathbf{v}} = \mathbf{v} - \mathbf{v_{avg}}$, where $\mathbf{v}$ represents the complete velocity of a node with respect to the earth fixed frame and includes the rotational velocity components, and $\mathbf{v_{avg}}$ denotes the time-varying mean velocity components in surge, sway and due to rotation in yaw, given by ($u_{avg} - y\dot{\Psi}_{avg}$, $v_{avg} + x\dot{\Psi}_{avg}$, 0). The variables u_{avg}, v_{avg} and $\dot{\Psi}_{avg}$ denote the average surge, sway and yaw velocities, respectively. The averaging is done over an encounter wave period.

The mixed boundary value problem (Equations 1 - 4) is solved for the perturbation potentials ϕ_R and ϕ_D and their derivatives. In the present work, a source distribution technique is used. De-

Main Particular	KCS	KVLCC2
$Type$	$Containership$	$VLCC$
L	230 m	320 m
B	32.18 m	58 m
T	10.8 m	20.8 m
C_B	0.65	0.81
Fn	0.33	0.142

Table 1: Main Particulars of Analyzed Vessels (Full Scale)

singularised sources are placed above the free surface nodes and constant strength panels are used on the body.

The pressure at any point on the hull surface is calculated using Bernoulli's equation as shown below

$$\frac{p}{\rho} + \left.\frac{\partial \phi}{\partial t}\right|_e + \frac{1}{2}\nabla\phi \cdot \nabla\phi + gz = 0 \tag{5}$$

where $\left.\frac{\partial \phi}{\partial t}\right|_e$ represents the temporal derivative of the perturbation potential with respect to the earth fixed frame. A direct formulation is used to set up a BVP for this term, often called the "acceleration potential". Details of this formulation can be found in [8]. The pressure is integrated over the hull surface to obtain the hydrostatic and external hydrodynamic forces and moments acting on the hull at a given instant in time. In the present study, all forces are computed by integrating the corresponding pressure components on the intersection surface of the exact body position and the incident wave surface. The body accelerations are computed by solving a fully nonlinear 6-DOF equations of motion solver. A 4^{th} order Adams-Bashforth-Moulton predictor-corrector scheme is used to time-step the equations of motion and solve for the evolution of the free surface.

3 RESULTS AND DISCUSSION

The simulations were set up and performed for two modern hull forms shown in Table 1. The KCS represents a high speed containership, and the KVLCC2 a slow speed large crude carrier. Based on convergence studies, a time step size of $T_e/100$ is used, T_e being the encounter period. On average, a total of 50 body panels are used for a full station. The free surface extends to 4 wavelengths on each side of the body, including 2 for the numerical beach, and the simulations are run for 15 encounter periods to obtain steady state solutions. The origin of the body frame is at the intersection of the calm water plane and the center-plane at the midship.

The response amplitude operators in head seas were extracted using an FFT of the force time history over the last two encounter wave periods. The comparisons with available experiments and other codes are shown in Figures 2 and 3. The predictions compare very favourably compared to the experiments with a general tendency to slightly over-estimate heave and show excellent agreement for pitch. The experimental data for the KCS was obtained from [9], and for the KVLCC2 from [10] (experiment 1) and CEHIPAR [11] (corresponding to experiment 2). Better agreement is seen for the KCS in heave and pitch compared to the linear frequency domain code MDLHydroD ([12]). For the KVLCC2, the 3D Rankine source based BEM ([11]) shows better agreement for heave at larger wavelengths, whereas the present body-exact computations are able to better capture the pitch motions throughout the frequency range.

REFERENCES

[1] Beck, R. F., and Reed, A. M. Modern computational methods for ships in a seaway. In *Transactions SNAME* (2001), pp. 1–51.

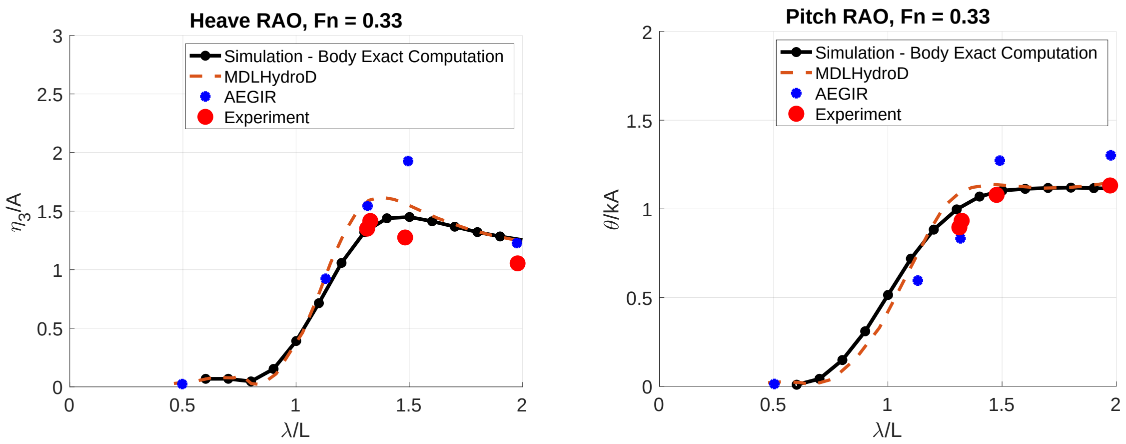

Figure 2: Vertical Plane Motions of KCS in Head Seas.

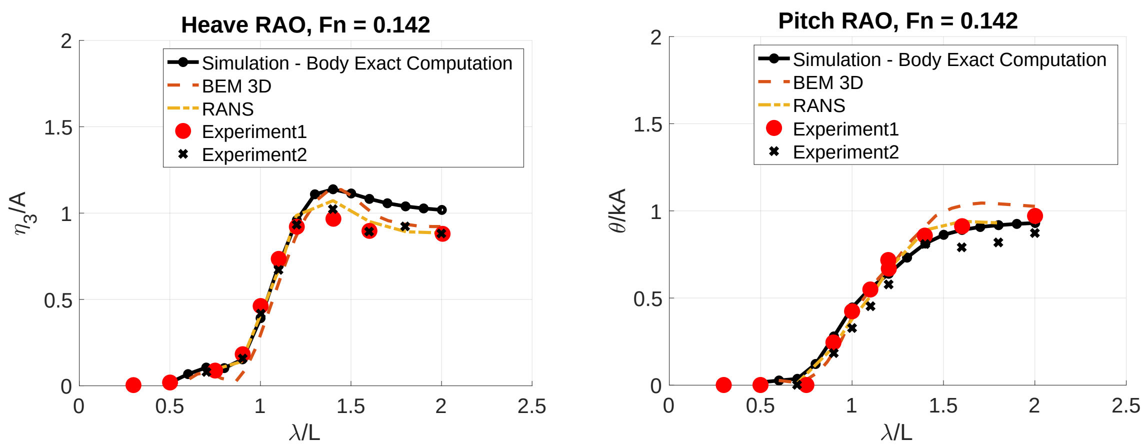

Figure 3: Vertical Plane Motions of KVLCC2 in Head Seas.

[2] Pawlowski, J. S., and Bass, D. W. 1991. *A Theoretical and Numerical Model of Ship Motions in Heavy Seas*. *SNAME Transactions 9*, 319–352.

[3] Huang, Y., and Sclavounos, P. D. 1998. *Nonlinear Ship Motions*. *Journal of Ship Research 42*, 120–130.

[4] Bandyk, P. J. 2009. *A body-exact strip theory approach to ship motions computations*. PhD thesis, The University of Michigan, Department of Naval Architecture and Marine Engineering.

[5] Subramanian, R., and Beck, R. F. 2015. *A time-domain strip theory approach to maneuvering in a seaway*. *Ocean Engineering 104*, 107–118.

[6] Subramanian, R., and Beck, R. F. An improved body-exact method to predict the maneuvering of ships in a seaway. In *Proceedings of the 38th International Conference on Ocean, Offshore& Arctic engineering, Glasgow, Scotland, UK* (2019), ASME.

[7] Subramanian, R., and Beck, R. F. Effect of wave height on maneuverability of ships in waves. In *SNAME Maritime Convention, Tacoma, Washington, USA* (2019), SNAME.

[8] Subramanian, R. 2012. *A time domain strip theory approach to predict maneuvering in a seaway*. PhD thesis, The University of Michigan, Department of Naval Architecture and Marine Engineering.

[9] Simonsen, C. D., Otzen, J. F., Joncquez, S., and Stern, F. 2013. *EFD and CFD for KCS heaving and pitching in regular head waves*. *Marine Science Technology 18*, 435–459.

[10] Park, D., Lee, J., and Kim, Y. 2015. *Uncertainty analysis for added resistance experiment of KVLCC2 ship*. *Ocean Engineering 95*, 143–156.

[11] Lyu, W., and Moctar, O. 2017. *Numerical and experimental investigations of wave-induced second order hydrodynamic loads*. *Ocean Engineering 131*, 197–212.

[12] Guha, A., and Falzarano, J. 2015. *The effect of hull emergence angle on the near field formulation of added resistance*. *Ocean Engineering 105*, 10–24.