A very simple park interaction factor

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Highlights An approximate q-factor is derived, and compared to the exact q-factor, computed by multiple scattering theory. The simple q-factor can be used to get a quick estimate of the park interaction in point-absorber wave energy parks, without carrying out any hydrodynamic computation for the park. The approximate method is evaluated as function of wave parameters and number of interacting devices.

1 Introduction

There is a vast literature on the wave-structure interaction and energy absorption in wave energy parks. Various analytical and numerical methods have been developed to solve the wave field of scattered and radiated waves within arrays of floating bodies, to analyse aspects such as optimal configurations, annual energy absorption, or power fluctuations [1–5]. The most common approach when studying large parks is to solve the fluid-structure interaction within the assumptions of linear potential flow theory, but extensions to higher non-linear interactions has also been developed. In many applications, such as economic or reliability optimization of large wave energy systems, thousands of iterations are needed to reach convergence [6]. In such situations, it is not feasible to carry out detailed analysis of the hydrodynamic interaction within the park during each iteration.

The q-factor, defined as the ratio between the total energy and the energy absorbed by the N WECs in isolation, is often used to evaluate the interaction between the devices. The simplest approach is assuming $q = 1$, i.e., to simply neglect all interactions between the wave energy converters (WECs) and assume that the total energy absorption is $P_{\text{tot}} = NP_1$, where N is the number of WECs in the park, and P_1 is the average energy absorption of a single isolated WEC in that sea state. This is often an overestimation of the park performance, as the interaction within wave energy arrays is typically destructive, at least when averaged over different sea states and wave directions, which is the realistic approach.

Here, another approach is presented. A very simple, approximate park interaction q-factor is derived. Using only knowledge of the energy absorption of a single WECs, as well as the number of WECs in the park and the size of the park, it can be used to estimate the total energy absorption of the park (the array layout is not required as an input). As compared to the simplest assumption of neglecting interaction, this approach typically gives an underestimation for the energy production, as it only includes the destructive interactions between WECs, and not the positive. It can thus be used as a lower bound for the performance. The approximate q-factor is compared to the exact q-factor, computed using multiple scattering theory. For a range of park configurations, with different number of WECs and different array layouts, the approximate method produces similar results.

Acknowledgedly, the approximate q-factor does not provide any fundamentally new insights into the fluid dynamics in wave energy parks. For this, accurate analytical, numerical, and/or experimental methods are needed. The method merely provides a very simple tool to estimate the interaction and performance, and can be of great use in many applications where the accurate methods are not feasible.

2 Theory

Consider a wave energy park of N point-absorbing wave energy converters (WECs), each consisting of a cylindrical surface buoy of radius $R = 3$ m connected to a linear generator at the sea bed. From the buoy oscillation driven by the waves, the WEC will absorb energy P_1, which will depend on the sea state parameters, the WEC dimensions and mass, and power take-off (PTO) settings. In terms of the capture width ratio (CWR) τ, the energy absorption is written $P_1 = \tau DJ$, where $J = \rho g^2/(64\pi) H_s^2 T_e$ is the energy flux defined in terms of the significant wave height H_s and energy period T_e of the waves.

Due to the presence and motion of the WECs, the total wave field in the park will be a superposition of scattered and radiated waves of all buoys. To determine the total energy absorption, P_{tot}, the full hydrodynamic interaction between all bodies must be determined, which is a challenging problem for large parks.

The park interaction factor, or q-factor, is defined as

$$q = \frac{P_{\text{tot}}}{NP_1}. \tag{1}$$

In most realistic cases and for large parks, the q-factor is below 1, showing that the total energy absorption of the park is less than if the N WECs were operating in isolation. If the energy absorption for single WECs and the q-factor is known, the total power of the park can thus be computed according to Eq. (1). Here, the analytical multiple scattering method is utilized to compute P_{tot} in Eq. (1), using the numerical implementation of [7]. In this approach, hydrodynamic interaction between all WECs in the park is computed semi-exactly, within the limits of linear potential flow theory, up to truncation $\Lambda_z = 40$ for the vertical eigenfunctions (evanescent modes) and $\Lambda_\theta = 3$ for the angular eigenfunctions.

The approximate method is now introduced. Given a park of N WECs deployed in an ocean area $L_{\mathrm{park}} \times L_{\mathrm{park}}$, the park layout is assumed as $\sqrt{N}$ rows of WECs along the length of the park L_{park}, with $\sqrt{N}$ WECs in each row. We further assume that the power absorbed by row j is only affected by the energy absorbed by the rows preceding it (as measured in the direction of the waves), i.e. shadowing effects are included, but no positive interaction. The energy absorbed by the $\sqrt{N}$ devices in the first row perpendicular to the incident waves is then $\sqrt{N}\tau DJ$, where again, τ is the CWR. Since the energy absorbed by the first row is not available to the second row, the available energy flux to row 2 is $J(1-\tau D\sqrt{N}/L_{\mathrm{park}})$. Consequently, the energy absorbed by row 2 is approximately $\sqrt{N}\tau DJ(1-\tau D\sqrt{N}/L_{\mathrm{park}})$. In the same way, the energy absorbed by row j is approximately

$$P_{\mathrm{abs,row}\ j} = \sqrt{N}\tau DJ\left(1 - \frac{\tau D\sqrt{N}}{L_{\mathrm{park}}}\right)^{j-1}. \tag{2}$$

To this end, we introduce the parameter

$$s = 1 - \alpha\tau, \tag{3}$$

where $\alpha = D\sqrt{N}/L_{\mathrm{park}}$ is the ratio of the total device width in a row and the length of the row. The parameter s satisfies $0 < s < 1$, and for few devices in the park as well as for small CWR τ, $s \to 1$. The total energy absorbed by the park during each hour can then be obtained as a geometric sum over the $\sqrt{N}$ rows,

$$P_{\mathrm{tot}}^{\mathrm{approx}} = \sum_{j=1}^{\sqrt{N}} P_{\mathrm{abs,row}\ j} = \sqrt{N}\,\tau D\,J\sum_{j=1}^{\sqrt{N}} s^{j-1} = \sqrt{N}\,\tau D\,J\frac{1-s^{\sqrt{N}}}{1-s} = NP_1 q^{\mathrm{approx}} \tag{4}$$

where we identified the approximate q-factor in expression (4) as

$$q^{\mathrm{approx}} = \frac{1-s^{\sqrt{N}}}{(1-s)\sqrt{N}}. \tag{5}$$

When the number of devices in the park decrease, the park interaction tends to unity $q^{\mathrm{approx}} \to 1$, implying that the interaction between the devices can be neglected and the power can be computed as $P_{\mathrm{tot}}^{\mathrm{approx}} = NP_1$. It decreases with increasing number of devices N, with increasing CWR τ or device width D, and with decreasing park area. The park interaction in Eq. (5) thus behaves as expected: it acts as a destructive shadowing effect which reduces the total power of the park, an effect which grows with more densely populated parks, and it can be neglected when the WECs in the park are few or separated by large distances.

3 Results

Figure 1 shows the results for the approximate q-factor, as compared to the exact results, and the case of neglected park interactions ($q = 1$). For the exact model, both parks with a square layout $\sqrt{N} \times \sqrt{N}$ have been modelled, as well as parks with random layouts. In the random layouts, the positions of the N WECs have been chosen randomly, with a minimum distance of $4R$ between devices. The park size has been increased according to number of WECs, such that the average distance to the closest neighbour stays approximately the same (15.1–22.6 m). Each park with random layouts has been modelled three times, as the results will differ for different WEC coordinates. An example of a park layout is shown in Fig. 2.

The single WEC performance will be different in different sea states and for different PTO damping values Γ. The results are therefore shown for two different sea states, characterised by (H_s, T_p) values equal to (1 m, 7 s) and (2 m, 12 s), and damping $\Gamma = 320$ kNs/m and 570 kNs/m, respectively. The PTO damping values are approximately the optimal damping values for a single WEC corresponding to those sea states, but similar results are obtained also for non-optimal damping values. Note that the computation for the approximate q-factor does not require any computation of the hydrodynamic interaction in the park, and the results could therefore be computed up to infinitely many devices with no computational effort. To enable a comparison with the exact results, however, we have here restricted the figures to 100 WECs.

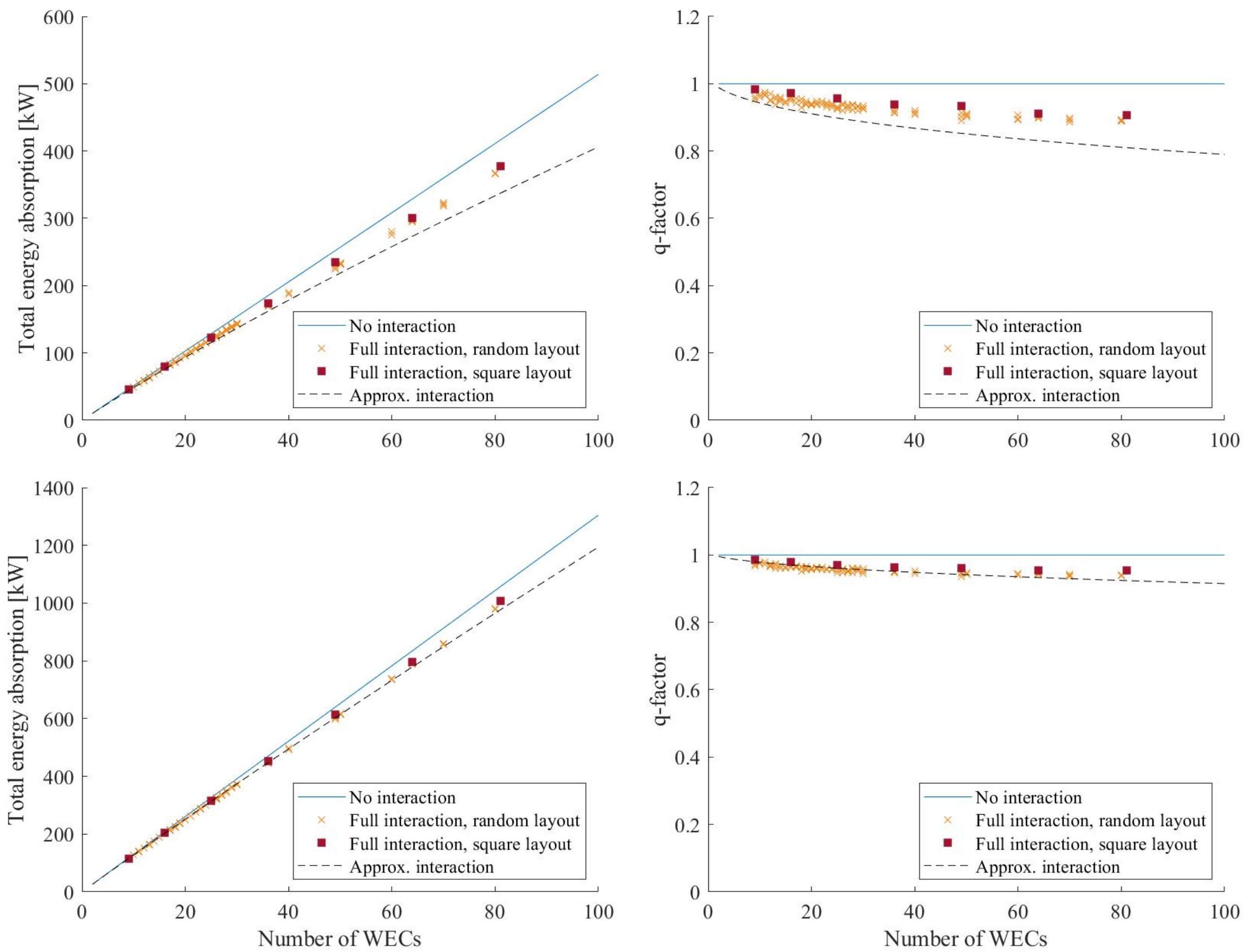

Figure 1: Performance of the q-factor as compared to the case of no interaction ($P_{\text{tot}} = NP_1$) and full interaction, computed using multiple scattering. Results shown for wave energy parks of increasing sizes (9-81 WECs). *Top row:* Sea state characterized by significant wave height $H_s = 1$ m and peak period $T_p = 7$ s, and a PTO damping of 320 kNs/m; *Bottom row:* significant wave height $H_s = 2$ m and peak period $T_p = 12$ s, with a PTO damping of 570 kNs/m.

To evaluate the approximate method further, and in particular the dependency on the wave climate, the square park with 81 WECs is evaluated in all the sea states corresponding to the annual wave climate at the WaveHub site, UK [8], with significant wave height H_s in the range 0.25–6.25 m and peak period T_p in the range 4.5–14.8 s. The results are shown in Fig. 2 in terms of the ratio $P_{\text{tot}}/P_{\text{tot}}^{\text{approx}}$ as function of the CWR for single WECs. The peak period of each individual sea state is indicated by the colorbar. Each sea state is simulated for a range of 9 different PTO dampings 190–750 kNs/m. The results corresponding to the optimal PTO damping for each sea state is highlighted with black circles.

As can be seen from the figure, the ratio between the exact and the approximate total energy is linearly dependent on the CWR. A low CWR gives an agreement close to 1, whereas higher CWR values correspond to a disagreement between P_{tot} and $P_{\text{tot}}^{\text{approx}}$ of up to 18.6%. The same relation holds true for the peak period; a better agreement for the total power is obtained in sea states with long peak periods, and worse agreement at short peak periods. This corresponds well to the results of Fig. 1, where the "range" between the approximate and no interaction lines is more narrow in the case with longer peak period (bottom plots), implying a closer agreement between the approximate and exact results. The linear relationship can be understood from the definition of the approximate interaction factor in Eq. (5). Using Eq. (1) and (4) for the exact and approximate energy absorption, their ratio becomes an expression that can be Taylor expanded around $s = 1$, or equivalently $\alpha\tau = 0$,

$$\frac{P_{\text{tot}}}{P_{\text{tot}}^{\text{approx}}} = \frac{q}{q^{\text{approx}}} = q\,\frac{(1-s)\sqrt{N}}{1-s^{\sqrt{N}}} = q\Big(1+\tfrac{1}{2}(\sqrt{N}-1)\alpha\tau+\tfrac{1}{12}(N-1)(\alpha\tau)^2+\tfrac{1}{24}(N-1)(\alpha\tau)^3+\mathcal{O}\big((\alpha\tau)^4\big)\Big). \quad (6)$$

Expanding also the q-factor as a linear function in the CWR, $q = q_0 + q_1\tau$, gives a polynomial expansion for the ratio in Eq. (6). For a park of $N = 81$ WECs, this expression becomes, to linear order,

$$\frac{P_{\text{tot}}}{P_{\text{tot}}^{\text{approx}}} = q_0\big(1+4\alpha\tau\big) + q_1\tau + \mathcal{O}\big(\tau^2\big) \quad (7)$$

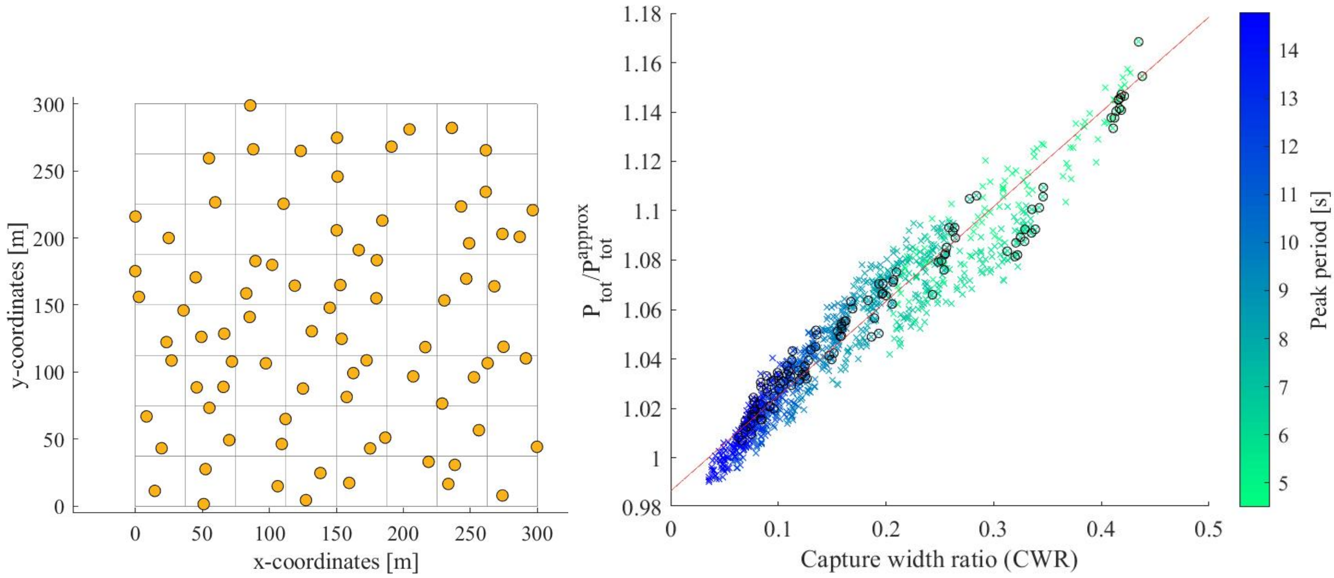

Figure 2: *Left:* Example of a random park layout of 81 WECs. In the square layout, the WECs are instead positioned on the 9×9 grid nodes. *Right:* Ratio of the total park power computed using full multiple scattering (P_{tot}) and the approximate method ($P_{\mathrm{tot}}^{\mathrm{approx}}$), for all sea states corresponding to an annual wave climate at the WaveHub site, UK. The ratio is shown as function of CWR for single WECs, and the peak period for each sea state is shown. The simulations corresponding to a PTO damping optimal for the particular sea state are highlighted with black circles. The simulations are carried out for a square park with 81 WECs. The expression for the plotted line is given in Eq. (7).

In Fig. 2, the line shown is the expression in Eq. (7), where the values q_0 and q_1 have been determined from fitting the data for the q-factor to a linear function of the CWR τ.

4 Conclusions

A simple expression for the park interaction q-factor has been derived, which can be used as an estimate of the total energy absorption of large wave energy parks. Only destructive interactions are taken into account, implying that the approximate interaction model can be used to estimate a lower bound for the absorbed energy.

The approximate q-factor and the resulting approximate energy absorption has been compared to exact values (computed using analytical multiple scattering) for many parks with 9-81 WECs, with both gridded and random layouts. To evaluate the performance as function of wave parameters, the comparison has been carried out for all sea states corresponding to an annual wave climate at the WaveHub site, UK. The results agree fairly well, with a better approximation obtained at higher wave periods and smaller CWR (of single devices). A linear relationship between the exact and approximate absorbed energy is obtained as function of the CWR.

It should be highlighted, that the strength of the approximate method lies not within its accuracy, but within its simplicity. Whereas the computation of hydrodynamic interactions within a large park of 100 WECs can require days of computational effort, even when parallel cores on HPC clusters are utilized, the estimate using the approximate q-factor is a simple multiplication of scalars.

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