Impulsive impact of a body submerged in an open container

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The concept of impulsive fluid/structure interaction widely used to study an initial stage of violent water impact flows, for which a strong couple between the nonlinear and unsteady effects may result in an extremely large hydrodynamic pressure and the force, respectively. Among the earliest work is that by Lagrange (1783) and Joukowskii (1884), in which the product of the density times the velocity potential is interpreted as the pressure impulse needed to suddenly impel a fluid from rest to its present velocity.

This concept has received much extension with application to aircrafts landing on water surface (von Karman, 1929), steep waves that suddenly hit coastal or marine structures (Cooker and Peregrine, 1995), impulsive motion of the submerged cylinder (Greenhow, 1987; Tyvand and Miloh, 1995) and impulsive sloshing in containers (Tyvand and Miloh, 2012), dam-break flows (Korobkin and Yilmaz, 2009), impact of blunt bodies onto a free surface (Howison, Ockendon and Wilson, 1991).

Impulsive impact of a body fully submerged into the half-space of the liquid was studied by (Semenov, Savchenko and Savchenko, 2021) using integral hodograph method. In the present study we extend the solution for the cases of containers or open channels of finite depth.

Boundary-value problem

A sketch of the physical domain is shown in figure 1(a). The body submerged below flat free surface is symmetric respect to Y-axis, therefore only half of the flow region is considered. Before the impact, $t = 0$, the body and the liquid are at rest. At time $t = 0^+$ the body is suddenly set into motion with acceleration a directed downwards such that, during infinitesimal time interval $\Delta t \to 0$, the speed of the body reaches the value $U = a\Delta t$. The problem of a rigid body moving in a fluid body is kinematically equivalent to the problem of a fluid body moving around a fixed rigid body with acceleration a of the bottom of the container or channel. We define a non-inertial Cartesian system of coordinates XY attached to the body at point A, and an inertial system of coordinates $X'Y'$ attached to the container. The body and the container are assumed to have an arbitrary shape, which can be defined by the slope of the boundary including the body and the container as a function of the arclength coordinate S, $\delta = \delta(S)$. The liquid is assumed to be ideal and incompressible, and the flow is irrotational. The gravity and surface tension effects are ignored.

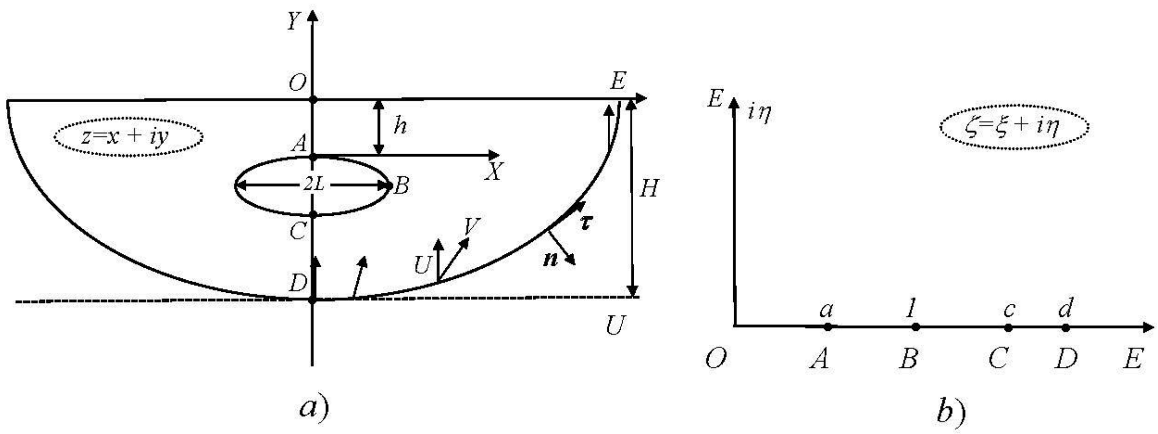

Figure 1. (a) The physical z-plane with the container and submerged body, (b) the parameter, or ζ–plane.

For two-dimensional inviscid, incompressible, irrotational flow we can introduce a complex potential $W(Z) = \Phi(x, y) + i\Psi(x, y)$ with $Z = X + iY$.

By integrating Bernoulli's equation

$$\frac{\partial \Phi}{\partial t}+\frac{p}{\rho}+\frac{|V|^2}{2}=\frac{p_a}{\rho}+\frac{U^2}{2}. \tag{1}$$

over the infinitesimal time interval $\Delta t \to 0$ and taking into account that the integral of the third term tends to zero, one can obtain

$$P=\int_0^{\Delta t} p dt=-\rho\Phi, \tag{2}$$

where P is the impulsive pressure. Here, $|V|$ is the velocity magnitude, p and p_a are the hydrodynamic pressure and the pressure on the free surface, respectively.

The vertical impulse force F_y is obtained by integrating the impulse pressure over the body surface,

$$F_y=-2\rho\int_{S_A}^{S_C}\Phi(S)\cos(n,y)\,dS=\mathrm{mL^2U}, \tag{3}$$

where S is the arc length coordinate along the body surface; S_A and S_C are the arcthlength coordinates of points A and C; m is the coefficient of the added mass. The multiplier "2" appears to account force acting on the whole body. Since we consider the body symmetric respect to y-axis, the horizontal impulse force equals zero. As it follows from equation (3), the coefficient of the added mass in the coordinate system XY is defined as

$$m=-2\rho\int_{S_A}^{S_C}\phi(s)\cos(n,y)\,ds. \tag{4}$$

In the paper Semenov, Savchenko and Savchenko (2021) it was shown that the added mass coefficients in the coordinate systems $X'Y'$ and XY are related as follows

$$m'=m-\rho A^* U, \tag{5}$$

where A^* is the cross-sectional area of the body.
The problem is to determine the velocity potential $\phi(x,y)$ immediately after the impact.

Conformal mapping

We introduce the complex potential $w(z)=\phi(x,y)+i\psi(x,y)$, where $\psi(x,y)$ is the stream function. To find the complex potential $w(z)$ directly is a challenge; therefore, we introduce an auxiliary parameter plane, or ζ -plane. We formulate boundary-value problems for the complex velocity function, dw/dz, and for the derivative of the complex potential, $dw/d\zeta$, both defined in the ζ -plane. Then the derivative of the mapping function is obtained as $dz/d\zeta=(dw/dz)/(dw/d\zeta)$, and its integration provides the mapping function $z=z(\zeta)$ relating the coordinates in the parameter and physical planes.

The boundary value problem for the complex velocity, dw/dz, is identical to the case of infinite depth of the liquid studied Semenov, Savchenko and Savchenko (2021). Therefore, we have

$$\frac{dw}{dz}=v_\infty\left(\frac{\zeta-a}{\zeta+a}\right)^{\frac{1}{2}}\left(\frac{\zeta-c}{\zeta+c}\right)^{\frac{1}{2}}exp\left[\frac{1}{\pi}\int_a^\infty\frac{d\beta_b}{d\xi}ln\left(\frac{\zeta-\xi}{\zeta+\xi}\right)d\xi-\frac{i}{\pi}\int_0^\infty\frac{dln\,v}{d\eta}ln\left(\frac{\zeta-i\eta}{\zeta+i\eta}\right)d\eta-i\frac{\pi}{2}\right], \tag{5}$$

where $\beta_b(\xi)=\delta(\xi)-\gamma(\xi)$ is the slope of the flow boundary as the function of the coordinate ξ, and $\gamma(\xi)$ is the angle between the velocity vector and the flow boundary. The body is considered to be fixed; therefore, the angle $\gamma(\xi)=\pi$ at interval $a\le\xi\le c$ corresponding to the body, and $\gamma(\xi)$ has to be determined at the interval $d\le\xi<\infty$ corresponding to the bottom surface. At the interval $c\le\xi\le d$, $\delta(\xi)=\frac{3\pi}{2}$ and $\gamma(\xi)=\pi$, i.e. $d\beta_b/d\xi=0$. The function $v(\eta)$ is the modulus of the velocity on the free surface just after the impact.

Now we formulate boundary value problem for the derivative of the complex potential, $dw/d\zeta$.

$$\vartheta(\zeta) = arg\left(\frac{dw}{d\zeta}\right) = arg\left(\frac{dw}{ds}\frac{ds}{d\zeta}\right) = arg\left(\frac{dw}{ds}\right) + \begin{cases} 0, \ \zeta = \xi, \\ -\frac{\pi}{2}, \ \zeta = i\eta, \end{cases} = \begin{cases} \pi, & 0 \le \xi \le d, \ \eta = 0, \\ \gamma(\xi), & d \le \xi < \infty, \ \eta = 0, \\ \frac{\pi}{2}, & 0 \le \eta < \infty, \ \xi = 0. \end{cases} + \begin{cases} 0, \ \zeta = \xi, \\ -\frac{\pi}{2}, \ \zeta = i\eta, \end{cases} \tag{6}$$

Equation (6) determines the argument of the complex function $dw/d\zeta$ on the whole fluid boundary, or on the real and imaginary axes of the ζ-plane. The integral formula (Semenov & Yoon (2009)),

$$F(\zeta) = K \exp\left[\frac{1}{\pi}\int_{\infty}^{0}\frac{\partial\vartheta}{\partial\xi}\ln\left(\varsigma^2-\xi^2\right)d\xi + \frac{1}{\pi}\int_{0}^{\infty}\frac{\partial\vartheta}{\partial\eta}\ln\left(\varsigma^2+\eta^2\right)d\eta + i\vartheta_\infty\right], \tag{7}$$

determines the complex function which satisfies boundary condition (6). Here, K is a real factor, $\vartheta(\zeta) = \arg[F(\zeta)]$, $0 < \xi < \infty, \eta = 0$ and $0 < \eta < \infty, \xi = 0$, $\vartheta_\infty = \vartheta(\zeta)_{|\zeta|\to\infty}$. We evaluate the integrals over each step change of the function $\vartheta(\zeta)$, and finally obtain the expression for the derivative of the complex potential in the ζ -plane as

$$\frac{dw}{d\zeta} = \frac{K}{\sqrt{d^2-\zeta^2}} exp\left[-\frac{1}{\pi}\int_0^\infty \frac{d\gamma}{d\xi} ln(\xi^2-\zeta^2)d\xi + i\theta_\infty\right]. \tag{8}$$

Free surface boundary conditions

The velocity generated by the impact is perpendicular to the free surface, or it is directed in y-direction. Accordingly, the argument of the complex velocity (5) is

$$arg\left(\left.\frac{dw}{dz}\right|_{\zeta=i\eta}\right) = -\frac{\pi}{2}, \quad 0 \le \eta \le \infty. \tag{9}$$

Taking the argument of the complex velocity from (5), we obtain the following integral equation respect to the function $\frac{d\ln v}{d\eta}$

$$\frac{1}{\pi}\int_0^\infty \frac{d ln\, v}{d\eta'} ln\left|\frac{\eta'-\eta}{\eta'+\eta}\right| d\eta' + \ tan^{-1}\frac{\eta}{a} + tan^{-1}\frac{\eta}{c} + \frac{1}{\pi}\int_a^\infty \frac{d\beta_b}{d\xi} 2tan^{-1}\frac{\eta}{\xi} d\xi = 0, \tag{10}$$

Equation (10) is the Fredholm integral equation of the first kind with the logarithmic kernel. The solution takes the form (Semenov, Savchenko and Savchenko, 2021).

$$v(\eta) = \sqrt{\eta^2+a^2}\sqrt{\eta^2+c^2}\exp\left(\frac{1}{\pi}\int_a^\infty \frac{d\beta_b}{d\xi}\ln(\eta^2+\xi^2)d\xi\right). \tag{11}$$

The parameters a, c and K are determined using the depth of submergence h, length of the low side BC and upper side AB of the plate.

Kinematic boundary condition on the solid surfaces

By integrating the derivative of the mapping function along the real axis of the parameter plane, we can determine the spatial coordinate along the body as a function of the parameter variable

$$s_b(\xi) = \int_0^\xi \left|\frac{dz}{d\zeta}\right|_{\zeta=\xi'} d\xi' \tag{12}$$

Since the function $\beta_b(s) = \delta(s) - \gamma(s)$ is known along the body and bottom surfaces, the function $\beta_b = \beta_b(\xi)$ is determined from the following integro-differential equation:

$$\frac{d\beta_b}{d\xi} = \frac{d\beta_b}{ds}\frac{ds_b}{d\xi}. \tag{13}$$

The angle $\gamma(s)$ along the bottom surface is determined from the condition

$$\beta\left|\frac{dw}{dz}\right|_{\zeta=\xi} \sin\gamma = 1, \quad d \le \xi < \infty. \tag{14}$$

Results

The added mass coefficients m' according to definition (5) are shown the in the Table for the flat plate submerged in the open channel

h/L	0.2	0.3	0.5	1	2	3	4	4.5	4.75
(H-h)/L	4.8	4.7	4.5	4	3	2	1	0.5	0.25
m'	2.162	2.302	2.514	2.828	3.009	3.119	3.386	4.374	4.706

Table 1. Added mass coefficient for the depth of the channel H/L=5.

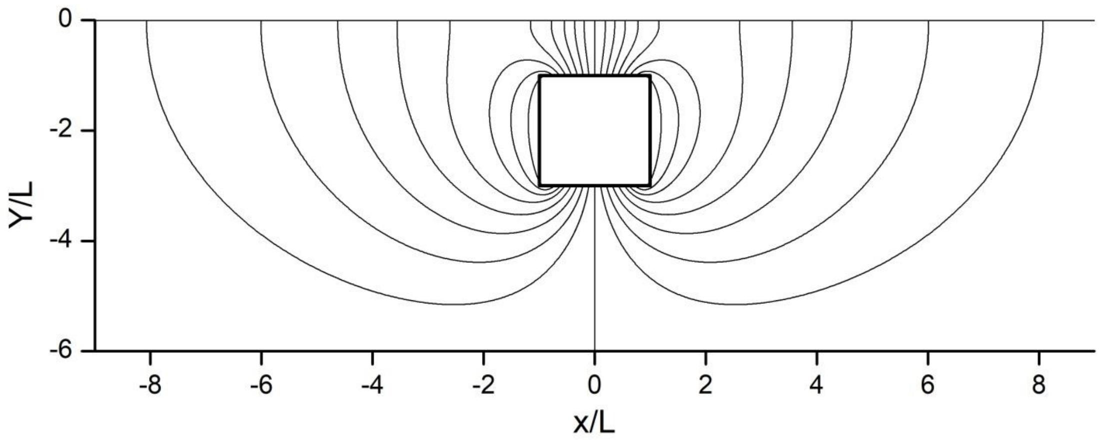

Figure 1. Streamline patterns for the impulsive motion of the square 2L$\times$ $2L$ in the coordinate system attached to the channel bottom.

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