Analysis of the pressure field in a breaking wave.

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Highlights

- numerical simulation of breaking wave in a sloshing tank,
- analysis of the onset of critical jets,
- spatial/temporal of the pressure field and its differential properties: gradient, Hessian matrix.

1) Introduction

The present abstract is concerned with highly nonlinear breaking wave in a rectangular tank. This kinematics is obtained by forcing the motion of the tank with a simple cyclic horizontal motion. As a result a plunging breaker is simulated. The same result could be probably obtained by simulating a dam-breaking case, starting from an initial potential energy and setting hence the total amount of available energy. However the experience shows that we can inject much more energy in the fluid thanks to the forced motion of the tank. In addition to the interest of being experimentally reproducible, the forced movement also has the advantage of injecting a quantity of energy much greater than the maximum potential energy reached during the simulation. The cyclic horizontal motion appears to be the simplest way to isolate some critical sequences when studying the loads on the walls of the tank (see Karimi *et al*, 2016). It is not the objective of the present study to assess the global or local loads. More interestingly it is observed in the present simulated highly nonlinear wave, the onset of critical jets that can appear suddenly along the free surface in the close vicinity of the nascent overturning crest leading to a plunging breaker. Some of these observations have been already commented in previous papers (see Scolan and Etienne, 2021).

The pionnering works by Longuet-Higgins (1980) provide us with mathematical tools to better anticipate the onset of highly accelerated flow when dealing with nonlinear free surface flows. Here in order to comment the mechanism of appearance of critical jets, it is proposed to analyze more deeply the temporal/spatial variations of the pressure. The principle is as follows; it is clear that a zero pressure gradient close to a free surface where the pressure itself vanishes, induce locally a high pressure gradient and consequently a high Lagrangian acceleration. However it is crucial to identify the mechanism according which a vanishing pressure gradient, particularly a local maximum, can occur close to the free surface. In that purpose higher order derivatives of the pressure are calculated, in particular its Hessian matrix and its eigenvalues and eigenvectors.

Among the critical jets the flip-through phenomenon (see Cooker and Peregrine, 1990) is already well known. Here other configurations are described and new types of critical jet are commented.

2) Governing equations

Basically, the equations are posed in the Potential Theory. The numerical software solves the fully nonlinear free surface boundary conditions. A desingularized technique avoids the discretization of an integral equation. The conformal mapping of the interior of the tank makes it possible to reduce the number of unknowns to the positions of the markers at the free surface only (see Scolan, 2010). The velocity potential is transported by the free surface markers. In the fluid the complex potential reads

$$F(z,t) = \sum_{j=1}^{N} q_j(t) G(w, W_j(t)) \tag{1}$$

where the complex coordinate w is the image of the complex coordinate z in the physical plane through the mapping function $w = -\cos\frac{\pi z}{L}$, L being the length of the rectangular tank. The intensity of the source # j is q_j and $W_j = g(Z_j)$ is the complex coordinate of the source # j in the transformed plane w. G is the complex potential of this source plus its mirror image with respect to the axis $\Im(w) = 0$; hence G meets the impermeability condition on the walls. The complex potential F describes the fluid motion in a coordinate attached to the tank. When the tank is in horizontal forced motion, additional terms appear. The pressure follows from Bernoulli equation written here in its complex form

$$F_{,t} + \frac{1}{2}|F_{,z}|^2 + \frac{p}{\rho} + \gamma(z,t) = 0 \tag{2}$$

where γ accounts for the acceleration fields (gravity and horizontal tank motion). Euler equation relates the pressure gradient to the velocity $\vec{u}$ as

$$\vec{u}_{,t} + (\vec{u}\cdot\vec{\nabla})\vec{u} + \frac{\vec{\nabla}p}{\rho} = \vec{\gamma} \tag{3}$$

where $\vec{\gamma}$ accounts for the acceleration fields (gravity and horizontal tank motion). The sum of the first two terms gives the Lagrangian acceleration $\frac{d\vec{u}}{dt}$. Due to the zero pressure condition at the free surface, the gradient of the pressure $\vec{\nabla}p$ is always oriented in the normal direction to the free surface, pointing into the fluid. In the present study, we also analyze the variations of the Hessian matrix that reads

$$\mathbf{H} = \begin{pmatrix} p_{,x^2} & p_{,xy} \\ p_{,xy} & p_{,y^2} \end{pmatrix} \tag{4}$$

This is a symmetric matrix and by using the following notations

$$f = \rho|F_{,z^2}|^2, \qquad A = \rho\left(F_{,z^3}\overline{F}_{,z} + F_{,z^2t}\right) = R + iI \tag{5}$$

the coefficients of the Hessian matrix are

$$p_{,x^2} = -f - R, \qquad p_{,y^2} = -f + R, \qquad p_{,xy} = I \tag{6}$$

The expressions of the eigenvalues λ and the eigenvectors $\vec{v}$ of the matrix $\mathbf{H}(x,y,t)$ then reads

$$\lambda_1 = -f + |A|, \qquad \lambda_2 = -f - |A|, \tag{7}$$

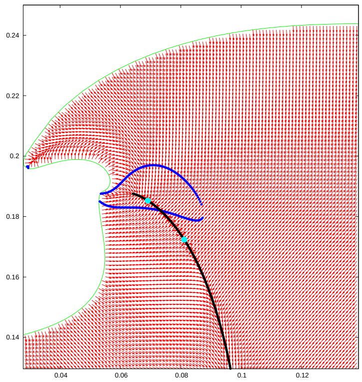
$$\vec{v}_1 = \frac{1}{\sqrt{2|A|}}\begin{pmatrix} sg(I)\sqrt{|A| - R} \\ \sqrt{|A| + R} \end{pmatrix}, \qquad \vec{v}_2 = \frac{1}{\sqrt{2|A|}}\begin{pmatrix} sg(I)\sqrt{|A| + R} \\ -\sqrt{|A| - R} \end{pmatrix} \tag{8}$$

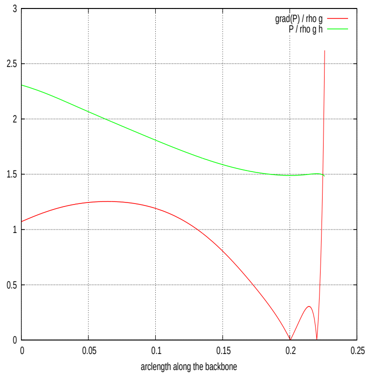
where $sg(I)$ denotes the sign of I. The Gaussian curvature Ω is the determinant of the matrix $\mathbf{H}$. This is the product of the eigenvalues, $\Omega = \lambda_1\lambda_2$. The inverse of each eigenvalue is also the curvature radius of the surface $p(x,y)$ along their corresponding principal directions defined by the eigenvectors. It is worth noting that one of the eigenvalues (denoted λ_2) is always negative meaning that the curvature of the surface $p(x,y)$ is "oriented down" along the direction of $\vec{v}_2$. λ_1 can change sign when $f = |A|$. As we are looking for zero of pressure gradient $p_{,x} = p_{,y} = 0$, we distinguish two cases. When both eigenvalues are negative, the Gaussian curvature is positive, indicating a local maximum. Otherwise, it is a saddle point. As we track the minimum of the pressure gradient over time and space, we shall seek the location where $||\vec{\nabla}p||$ is minimum or even vanishes. It is shown that this also occurs when $\vec{\nabla}p$ and $\vec{v}_1$ are parallel, leading to search for the change of sign of the following quantity

$$v = Ip_{,y} - (R + |A|)p_{,x} \tag{9}$$

2) Illustrative results

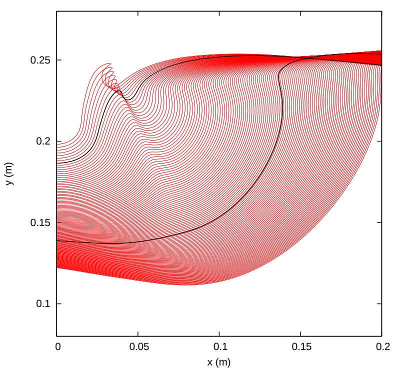
The locations where v changes of sign is illustrated with a black line in the following figure (left).

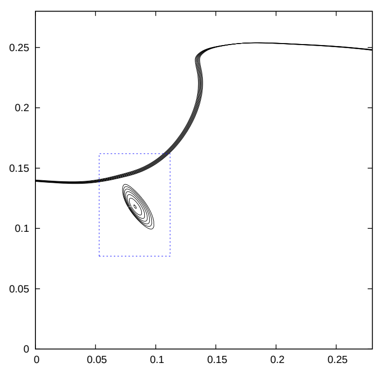




The vector field corresponds to the normalized pressure gradient. The blue line encloses the region of positive Gaussian curvature. The two light blue spots show the location of a saddle point and a local maximum of the pressure in the region where $\Omega > 0$. The black line $v = 0$ can be described with an arclength and in the present case, this line starts from the bottom. The right figure shows the spatial variation of $\frac{p}{\rho g h}$ and $\frac{||\vec{\nabla} p||}{\rho g}$ along the line $v = 0$. The pressure reaches a threshold $\frac{p}{\rho g h} \approx 1.5$ in the region $\Omega > 0$, then the pressure must vanish at the free surface, hence leading to a pressure gradient which can be greater than $\frac{||\vec{\nabla} p||}{\rho g} > 300$. This is the mechanism that makes appear and develop the critical jet.

The part of the free surface where high kinematics is expected is located between the left wall and the tip of the nascent overturning crest. When a plunging breaker fully develops, this region is oftenly called the barrel by the surfers. It is proven that the maximum acceleration and the maximum of velocity are observed in the barrel (see Yasuda *et al*, 1997). In this region several type of critical flows can appear. Before the barrel is well formed Peregrine (2003) shows that the flip-through appears to be the result of a competition between the run-up at the wall and the advancing crest front towards the wall. The free surface rather looks like a parabola whose curvature radius shrinks until the free surface flips leading to a small jet (see Cooker, 2009). At this instant the acceleration can reach very high values (more than ten thousand times the gravity). The shape of the free surface before this occurs is analyzed by Longuet-Higgins (2001). In particular he proposes to flatten the trough drawn by the free surface at the wall. As a consequence he observes a local growth that precedes the launch of a jet, the whole sequence is called the bazooka effect (see the fig. 6 in Longuet-Higgins, 2001). Actually this "bazooka" looks like a standard flip-through. In the present study the initial local growth is called a pre-bazooka effect. Indeed this local growth is the sign of appearance of a new type of jet as illustrated below





The left figure shows the successive free surface profiles for a case where a pre-bazooka appears. Two particular instants are emphasized with black lines. The first instant corresponds to the change of sign of the curvature radius along the free surface, hence announcing the local growth. Before this instant, it is observed that the Gaussian curvature Ω becomes positive at a single point inside the fluid just below the location where the pre-bazooka will appear. Then this region of positive Gaussian curvature increases until hitting the free surface. There is a strong correlation between the change of sign of the curvature radius along the free surface and the change of sign of the Gaussian curvature as shown in Scolan and Etienne (2021). The reason is $\vec{v}_2$ is oriented normally to the free surface as $\vec{\nabla} p$. Here the local change of sign of Ω announces an event as a premonitory sign. Unfortunately this is not observed in Longuet-Higgins' computations. One reason could be that not enough kinetic energy is available. The second instant of interest corresponds to the maximum acceleration reached during this simulation. This occurs in the trough on the left of the nascent inverted crest. In the present case this maximum acceleration does not exceed 400 times the gravity. There is common feature of all overturning crests, the main one like the small one in a critical jet; the crest turns towards the region where the Lagrangian acceleration is maximum. It is also noticeable that the free surface as a parabolic profile at the wall. As a consequence a flip-through is likely to occur. In the present case the small inverted crest (similar to a "tuft of hair" on the head of the wave) breaks before the onset of the flip-through. A parametrical analysis in terms of the mean water depth h shows an inversion of chronology; the flip-through occurs before the breaking inverted crest.

The present analysis the pressure field allows to revisit some recent studies concerning the occurrence of wave breaking (see Barthelemy *et al*, 2018 among others). This proposed criterion is the ratio of the horizontal fluid velocity at the wave crest to the phase velocity. This is a ratio of kinematic quantities and it is found that there is a threshold above which the wave breaks. This threshold varies slightly depending on the type of wave and on the type of bathymetry. It is proposed here that there could also exist a dynamic criterion which would link the Lagrangian acceleration to the pressure gradient. Preliminary numerical tests consists of a dam breaking case that leads to a soliton that breaks (or not). The quantities $||\frac{d\vec{u}}{dt}||$ and $||\vec{\nabla} p||$ are calculated along the line where $v = 0$ which links the maximum free surface elevation to the bottom of the tank. A maximum of $\rho||\frac{d\vec{u}}{dt}||/||\vec{\nabla} p||$ exists along the line $v = 0$. This maximum has a threshold depending on whether or not the wave breaks.

3) References

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