# Modelling design waves in a Numerical Wave Flume based on a higher-order Moving Particle Semi-Implicit Method


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## HIGHLIGHTS

- We present a numerical wave flume based on a higher-order Moving Particle Semi-Implicit Method (NWF-MPS).
- We introduce a novel artificial viscosity term to prevent particle clustering and to reduce spurious numerical oscillations.
- The model is used to successfully reproduce waves of engineering interest in linear and nonlinear regimes, using fewer particles than needed by other Lagrangian methods.

## 1 INTRODUCTION

The Moving Particle Semi-Implicit Method is a meshless Computational Fluid Dynamics (CFD) model based on a Lagrangian description of the fluid flow. In Lagrangian models, the fluid is discretised into computational particles, which act as calculations points where the hydrodynamic quantities (such as velocity and pressure) are calculated.

Applications of particle based models to wave dynamics date back to three decades ago, when the Smoothed Particle Hydrodynamics (SPH) method, originally developed in the 1970's for problems in astrophysics, was extended to solve free-surface flows [1]. The development of MPS started much later, around the beginning of this century, as an extension of the finite volume method [2]. MPS has several advantages as compared to SPH, such as a simplified approach to spatial discretisation and a lower sensitivity to the kernel choice [3].

Several authors have proposed MPS applications to water waves, including wave breaking, wave-ship interaction and landslide tsunamis (see [3, 4] and references therein). However, there is still lack of systematic MPS modelling of design waves in a numerical wave flume (NWF).

Here we report novel results stemming from the recent work of [3], in which a higher-order MPS model was developed to reproduce linear and nonlinear waves.

## 2 NUMERICAL MODEL

We consider a viscous and weakly compressible fluid flow governed by the continuity and Navier-Stokes equations, respectively

$$\frac{\mathrm{D}\rho}{\mathrm{D}t} + \rho\nabla\cdot\mathbf{u} = 0, \tag{1}$$

and

$$\frac{\mathrm{D}\mathbf{u}}{\mathrm{D}t} = -\frac{1}{\rho}\nabla P + \nu\nabla^2\mathbf{u} + \mathbf{g}. \tag{2}$$

In the latter, $\mathrm{D}/\mathrm{D}t$ is the Lagrangian derivative, $\nabla = (\partial/\partial x, \partial/\partial y)^T$ is the nabla operator, $\rho$ is density, $P$ is pressure, $\nu$ is kinematic viscosity and $\mathbf{g} = (0, -g)^T$, $g = 9.807\,\mathrm{ms}^{-2}$ is gravity. Following [5, 6, 7], a weakly compressible formulation is used for the density, so that

$$\rho = \rho^0 + \frac{\mathrm{d}\rho}{\mathrm{d}P}\,\mathrm{d}P, \tag{3}$$

where $\rho^0$ is the constant ambient density and $c = (\mathrm{d}P/\mathrm{d}\rho)^{1/2}$ is the speed of sound in water.

### 2.1 MPS Formalism

The MPS method solves a discretised form of the governing equations (1)–(2), calculated at particle $i$ at the $k$-th instant $t^k$. This is achieved by using a weighted average over all the neighbouring particles $j = 1, ..., N_i$, where the weight function is given by

$$w(r, r_e) = \begin{cases} r_e/r - 1 & \text{if}\, r < r_e \\ 0 & \text{if}\, r \geq r_e \end{cases}. \tag{4}$$

In the latter, $r_e$ is an effective radius of interaction. Summing the weight functions calculated at all the neighbouring particles of the $i$-th target particle gives the particle number density

$$n_i = \sum_{j \neq i}^{N_i} w\left(r_{ji}, r_e\right), \tag{5}$$

where $\mathbf{r}_{ji} = \mathbf{r}_j - \mathbf{r}_i$, $r_{ji} = |\mathbf{r}_{ji}|$, and $\mathbf{r}_i$ is the position of particle $i$.

### 2.2 Numerical Solution

The governing equations are then solved numerically using a predictor-corrector scheme, where the $i$th particle velocity $\mathbf{u}_i$ is first calculated at an intermediate step $t^*$, without considering the pressure gradient:

$$\mathbf{u}_i^* = \mathbf{u}_i^k + \left(\nu \left\langle \nabla^2 \mathbf{u} \right\rangle_i^k + \mathbf{g}\right) \Delta t. \tag{6}$$

The value is then corrected by accounting for the pressure contribution:

$$\mathbf{u}_i^{k+1} = \mathbf{u}_i^* - \frac{1}{\rho} \left\langle \nabla P \right\rangle_i^{k+1} \Delta t, \tag{7}$$

where $\Delta t$ is the time discretisation. Eq. (7) is fully determined once the pressure is known. The pressure solves the Poisson equation

$$-\frac{1}{\rho_0} \left\langle \nabla^2 P \right\rangle_i^{k+1} + \frac{\alpha}{\Delta t^2} P_i^{k+1} = \frac{n_i^* - 2n_i^k + n_i^{k-1}}{n^0 \Delta t^2} + B \frac{n_i^k - n_i^{k-1}}{n^0 \Delta t} + \Gamma \frac{n_i^k - n^0}{n^0}, \tag{8}$$

where $\alpha = 1/(\rho c^2)$, $B = 500$, $\Gamma = 50,000$ as recommended by [8]. In (8) the Laplacian is given by

$$\left\langle \nabla^2 P \right\rangle_i^k = \frac{4}{\lambda^0 n^0} \sum_{j \neq i} \left(P_j^k - P_i^k\right)\, w\left(r_{ji}, r_e\right), \tag{9}$$

where

$$\lambda^0 = \frac{1}{n^0} \sum_{j \neq l} \left(r_{jl}^0\right)^2\, w\left(r_{jl}^0, r_e\right) \tag{10}$$

is a weighted average at time $t = 0$, and $l$ is an internal fluid particle. Substitution of (9) in (8) yields a linear system for the $P_i^k$'s. Once the pressure is known from solving (8), the pressure gradient in (7) is calculated with the minimum pressure $\hat{P}_i^{k+1}$ in the neighbourhood of the $i$-th particle:

$$\langle \nabla P \rangle_i^{k+1} = \mathbf{C}_i^{-1} \cdot \left( \frac{1}{n^0} \sum_{j \neq i}^{N_i} \frac{P_j^{k+1} - \hat{P}_i^{k+1}}{\left(r_{ji}^*\right)^2} \mathbf{r}_{ji}^* \, w\left(r_{ji}^*, r_e\right) \right), \tag{11}$$

where the $2 \times 2$ matrix $\mathbf{C}_i$ is given by

$$\mathbf{C}_i = \frac{1}{n^0} \sum_{j \neq i}^{N_i} \frac{\mathbf{r}_{ji}^*}{r_{ji}^*} \otimes \frac{\left(\mathbf{r}_{ji}^*\right)^T}{r_{ji}^*} \, w\left(r_{ji}^*, r_e\right), \tag{12}$$

see [3, 8, 9]. To prevent particle clustering and tensile instability, we add an additional viscous term to the predictor step:

$$\left\langle \frac{\mathrm{D}\mathbf{u}}{\mathrm{D}t} \right\rangle_i^k = \frac{\delta\, c}{n^0} \sum_{i \neq j}^{N_i} \left( \frac{\mathbf{u}_{ji}^k \cdot \mathbf{r}_{ji}^k}{\left(r_{ji}^k\right)^2 + 0.01\, r_e^2} \right) \left( \frac{r_e}{r_{ji}^k} \right)^2 \frac{\mathbf{r}_{ji}^k}{r_{ji}^k}, \tag{13}$$

where $\delta$ is an artificial viscosity parameter.

Appropriate boundary conditions must be applied on the free surface and at the solid boundaries. On the free surface, we request that the pressure be atmospheric, i.e. $P = 0$. In other words, we only consider the excess pressure over the atmospheric value. On the solid boundaries, we request a no-slip condition, so that the velocity is zero at boundary particles. Waves are generated using a numerical wavemaker at the left end of the flume, based on a dynamic boundary method (DBM). For more details on the implementation of those conditions into the computational algorithm, we refer interested readers to [3].

## 3 APPLICATIONS

The NWF-MPS model has been validated with respect to available analytical and experimental data. An extensive validation exercise, including detailed convergence studies, is available in [3]. Here we report an application of the above model to nonlinear wave propagation in a channel. More applications, including random seas and highly nonlinear dynamics, will be presented at the Workshop.

Figure 1 shows the time series of second-order Stokes waves generated in the flume, compared with Madsen's analytical formula [10]. The wavemaker generates a first-order component and a second-order bound wave, which is known to steepen the crests and broaden the troughs. The flume is 15 m long and 0.64 m deep, the wave height is 0.15 m and the period is 2 s. To prevent unwanted reflections from the end wall of the flume, a relaxation zone has been introduced at the end of the tank. The total number of particles is 42,236 and the time step is 0.001 s.

There is very good agreement between the analytical and numerical data. The correlation coefficient is $R = 0.988$ and the root mean square error is $E = 0.01$. We remark that a similar configuration was studied by [11] with an SPH solver. Interestingly, [11] obtained similar convergence results, but with an order $O(10^5)$ particles, as opposed to $O(10^4)$ needed by the NWF-MPS model.

## 4 CONCLUSIONS

We presented a novel Numerical Wave Flume based on a higher-order Moving Particle Semi-Implicit method (NEW-MPS). A peculiar aspect of the model is the addition of an artificial

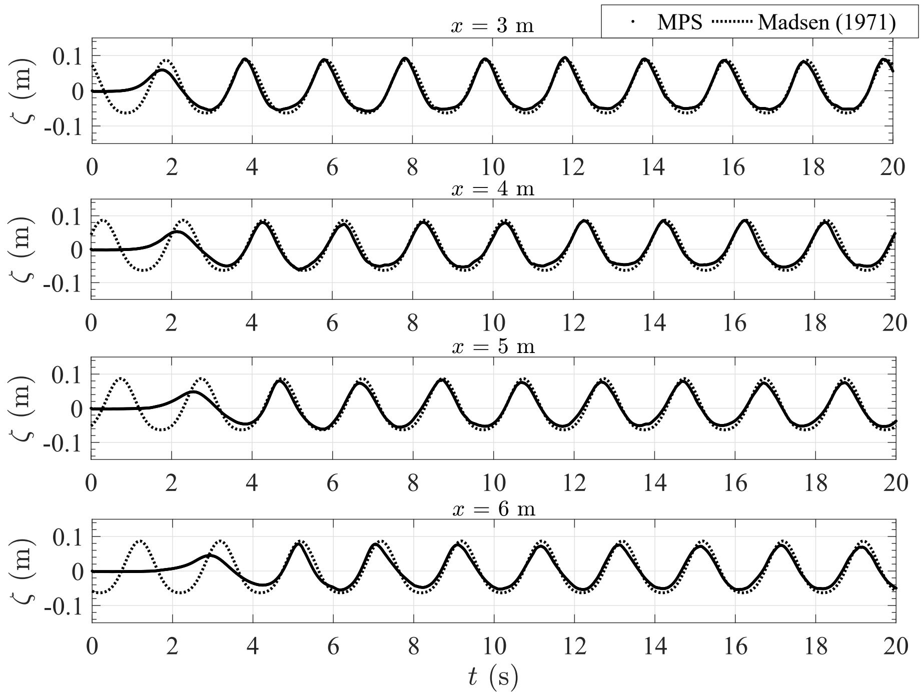


Figure 1: Comparison between MPS data and Madsen's analytical formula

viscosity term that prevents tensile instability and particle clustering, thus allowing to propagate waves even at large times. The model is able to reproduce design waves of engineering interest, such as second-order Stokes waves. Further examples on random sea states and wave-structure interactions will be presented at the Workshop.

## REFERENCES


[1] Monaghan, J. 1994. *Simulating free surface flows with SPH. J. Comput. Phys. 110*, 399–406.

[2] Koshizuka, S., and Oka, Y. 1996. *Moving-particle semi-implicit method for fragmentation of incompressible fluid. Nucl. Sci. Eng. 123*, 421–434.

[3] Renzi, E., and Dias, F. 2021. *Application of a Moving Particle Semi-Implicit Numerical Wave Flume (MPS-NWF) to model design waves. Coast. Eng.*, 104066.

[4] Gotoh, H., and Khayyer, A. 2018. *On the state-of-the-art of particle methods for coastal and ocean engineering.* Coast. Eng. J. 60( 1), 79–103.

[5] Renzi, E., and Dias, F. 2014. *Hydro-acoustic precursors of gravity waves generated by surface pressure disturbances localised in space and time. J. Fluid Mech. 754*, 250–262.

[6] Renzi, E. 2017. *Hydro-acoustic frequencies of the weakly compressible mild-slope equation. J. Fluid Mech. 812*, 5–25.

[7] Michele, S., and Renzi, E. 2020. *Effects of the sound speed vertical profile on the evolution of hydroacoustic waves. J. Fluid Mech. 883*, A28.

[8] Kondo, M., and Koshizuka, S. 2011. *Improvement of stability in moving particle semi-implicit method.* Int. J. Numer. Meth. Fluids 65( 638–654).

[9] Koshizuka, S., Shibata, K., Kondo, M., and Matsunaga, T. 2018. *Moving Particle Semi-Implicit Method.* Academic Press, United Kingdom.

[10] Madsen, O. 1971. *On the generation of long waves. J. Geophys. Res. 76*, 8672—-8683.

[11] Altomare, C., Dominguez, J., Crespo, A., Gonzalez-Cao, J., Suzuki, T., Gomez-Gesteira, M., and Troch, P. 2017. *Long-crested wave generation and absorption for SPH-based DualSPHysics model. Coast. Eng. 127*, 37–54.