Enhanced Wave-Field Reconstruction Based on Stochastic Characteristics of Shadowing Effects

Yoon-Seo Nam[1], Jae-Hoon Lee[1], Yuming Liu[2] and *Yonghwan Kim[1]

[1] Department of Naval Architecture and Ocean Engineering, Seoul National University, Seoul, Korea
[2] Department of Mechanical Engineering, Massachusetts Institute of Technology, Cambridge, MA, USA
*yhwankim@snu.ac.kr

1 INTRODUCTION

In order to analyze the performance of ships and offshore platforms on a real-time basis, the phase-resolved prediction for a surrounding wave field is required. The sensing mechanism of a marine radar involves modulation effects such as shadowing which engender non-physical effects in the radar image. Therefore, post-processing, that is, the wave-field reconstruction should be performed to scale the image intensity and to remove the non-physical components. In the present study, we propose an enhanced reconstruction method based on the stochastic characteristics of the shadowing effect.

2 THEORETICAL BACKGROUNDS

A radar beam with a slope (μ) is assumed to be emitted from the radar that is located far away from the origin of the coordinate system. In the coordinate system (x, z), the x-axis is aligned with the radar ray and z=0 indicates the mean sea surface. The wave elevation (ζ) and wave slope (q) are considered independent zero-mean Gaussian random processes with the standard deviation of σ and w, respectively. The spatial autocorrelation function of the wave elevation is denoted by *acf*. Moreover, the random variables at the origin (x=0) are abbreviated as (ζ_0, q_0), and variables at x are abbreviated as (ζ_1, q_1) to simplify expressions. The coordinate system and definitions are summarized in Fig. 1. Then, the shadowing function, $S(\zeta_0, q_0, \mu; \sigma, w)$ is defined with respect to the given variables, which represents the probability that the shadowing will not occur at the origin. The general form of the shadowing function is as follows:

$$S(\zeta_0, q_0, \mu; \sigma, w) = \begin{cases} \exp\left[-\int_0^\infty g(x)\,dx\right] & \mu \ge q_0 \\ 0 & \mu < q_0 \end{cases} \tag{1}$$

where $g(x)$ indicates the conditional probability that the waves between [x, $x+dx$] interfere with the radar ray when no shadowing occurs between [0, x]. The conditional shadowing probability can be expressed by introducing a joint Gaussian probability density function, $P_4(\zeta_0, \zeta_1, q_0, q_1)$, such that:

$$g(x) = \frac{\int_\mu^\infty (q_1 - \mu) P_4(\zeta_0, \zeta_0 + \mu x, q_0, q_1)\,dq_1}{\int_{-\infty}^\infty \int_{-\infty}^{\zeta_0 + \mu\tau} P_4(\zeta_0, \zeta_1, q_0, q_1)\,d\zeta_1 dq_1}. \tag{2}$$

Particularly, $g(x)$ can be calculated with or without considering the spatial autocorrelation effects. The details about the evaluation of $g(x)$ without the correlation effects can be found in [1], and an enhanced computation with the correlation is presented in [2]. Based on the derived conditional probabilities in the previous studies, $S(\zeta_0, q_0, \mu; \sigma, w)$ can be computed.

Further stochastic characteristics independent of the standard deviation of wave elevations (σ) can be derived from the shadowing function. The Smith function, $S(\mu; w)$, which denotes the probability of not shadowing, is defined by averaging the shadowing function over (ζ_0, q_0):

$$S(\mu; w) = \int_{-\infty}^\infty \int_{-\infty}^\infty S(\zeta_0, q_0, \mu; \sigma, w) \frac{1}{2\pi\sigma w} \exp\left(-\frac{\zeta_0^2}{2\sigma^2} - \frac{q_0^2}{2w^2}\right) d\zeta_0 dq_0. \tag{3}$$

For a sequence of consecutive radar images, the illumination ratio (the ratio of not shadowed time to the entire measurement duration at a certain location) can be obtained. Hence, it is possible to estimate the mean surface slope in each azimuthal direction ($w_{est}(\theta)$) from the radar images by fitting the illumination ratio with the Smith function. Details about the mean surface slope estimation are presented in [3].

Based on the estimated mean surface slope and the relationship between sea state parameters, we propose the following estimation for the significant wave height, which is required to scale the image intensity according to the total energy of an ocean wave field:

$$H_S = \frac{g w_{total} T_4^2}{\pi^2} \tag{4}$$

where $w_{total} = \sqrt{w_{est}(\theta)^2 + w_{est}\left(\theta + \frac{\pi}{2}\right)^2}$, $T_4 = 2\pi\left(\frac{m_0}{m_4}\right)^{1/4}$.

Here, m_n indicates the n^{th} spectral moment computed by the FFT analysis on the radar image sequence.
Furthermore, it is possible to estimate the expected variance (energy) of the image intensity. In the present study, the Smith variance is proposed, which represents the ratio between the expected variances of the shadowed (ζ_s) and original wave-field (ζ). Explicit formulation of the Smith variance is as follows:

$$V(\mu;w) = \frac{\mathrm{var}(\zeta_s)}{\mathrm{var}(\zeta)} = \frac{1}{\sigma^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \zeta_0^2 S(\zeta_0, q_0, \mu_0; \sigma, w)\frac{1}{2\pi\sigma w}\exp\left(-\frac{\zeta_0^2}{2\sigma^2} - \frac{q_0^2}{2w^2}\right) d\zeta_0 dq_0 \tag{5}$$

where $\zeta_s(x,t) = \begin{cases} \zeta(x,t) & \textit{no shadowing} \\ 0 & \textit{shadowing} \end{cases}$.

By adopting the mean-shift modification method proposed by Zinchenko [4], the radar image can be modified to resemble the shadowed wave field. Subsequently, the calibration of image intensity based on the Smith variance is conducted so that the energy level could be uniform in the entire measurement domain such as an ocean wave field. For this purpose, the radar image intensity (ρ) is divided by the square root of the expected variance at each location (r, θ) as follows:

$$\rho_c(r,\theta) = \frac{\rho(r,\theta)}{\sqrt{V(\mu(r); w_{est}(\theta))}} \qquad \text{where } \mu(r) = r/h_r. \tag{6}$$

Here, h_r is the radar height.

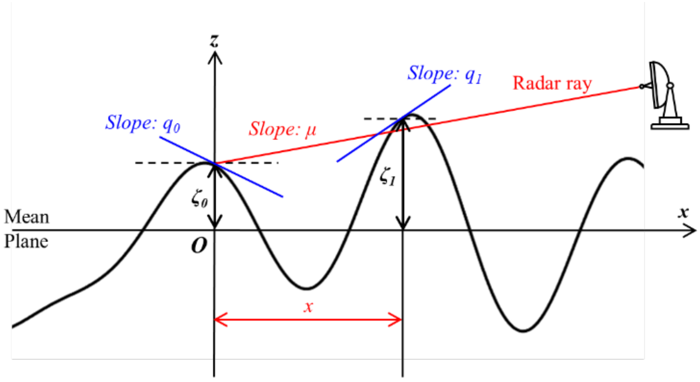
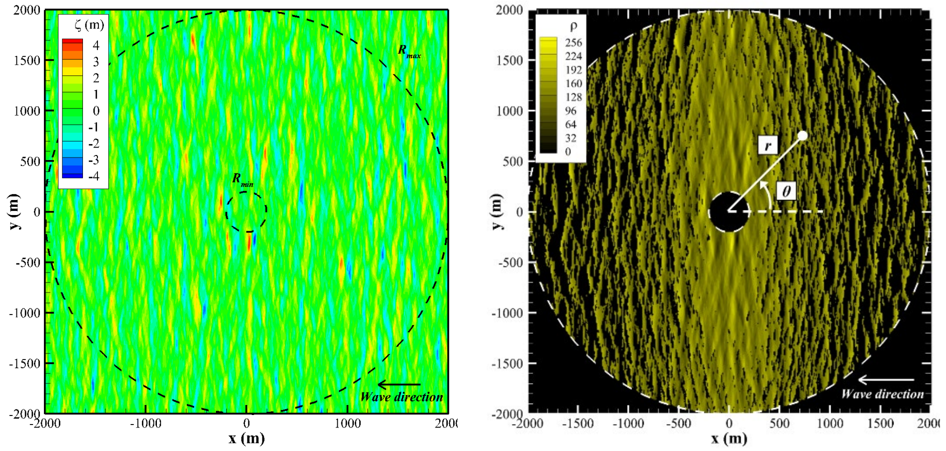


Fig. 1 Coordinate system and definitions

3 COMPUTATION RESULT

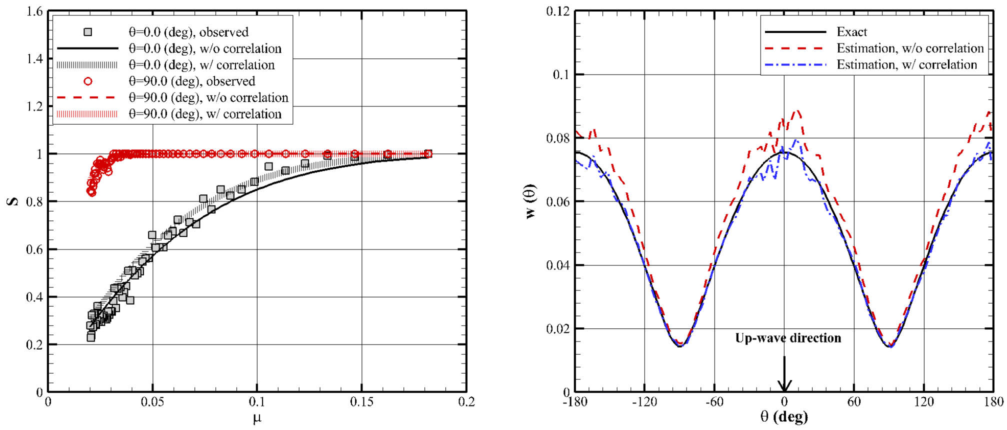
Synthetic radar images are generated to validate the proposed wave-field reconstruction method. To this end, a short-crested wave data of Beaufort scale 7 (H_S=4.0 m and T_{mean}=7.7 s) is synthesized based on the linear superposition of random wave components. The main propagation direction and the spreading angle are χ_M=180.0 deg and χ_S=60.0 deg, respectively. Moreover, the assumed minimum and maximum sensing radius are $R_{\min}$=200.0 m and $R_{\max}$=2000.0 m with the spatial resolution dx=dy=10.0 m. The length of the time window is T=100.0 s and the time interval between two successive radar images is set to dt=1.0 s. Subsequently, the geometrical shadowing effect is reflected to generate the synthetic image intensity with the radar height of h_r=40 m. The intensity of the shadowed point is set to be zero, whereas the non-shadowed points are rescaled into an 8-bit grey scale (0-255). The generated synthetic wave data and radar image are shown in Fig. 2.
First, the mean surface slope is estimated by fitting the illumination ratio with the Smith function as presented in Fig. 3(a). The estimated mean surface slopes with and without considering the spatial autocorrelation effect

are compared in Fig. 3(b). The exact mean surface slope is computed based on the analytic integration of a given wave spectrum. As shown in the figure, the mean surface slope estimation is more accurate when the spatial autocorrelation effects are taken into account for the derivation of the Smith function. Accordingly, the accuracy of the H_S estimation is also improved due to the correlation effects (without correlation: $H_{S,est}/H_{S,exact}$=1.178, with correlation: $H_{S,est}/H_{S,exact}$=1.051).



(a) Synthetic wave data (b) Synthetic radar image

Fig. 2 Synthetic wave-field and radar image: Short-crested waves of Beaufort scale 7 (H_S=4.0 m, T_{mean} = 7.7 s)

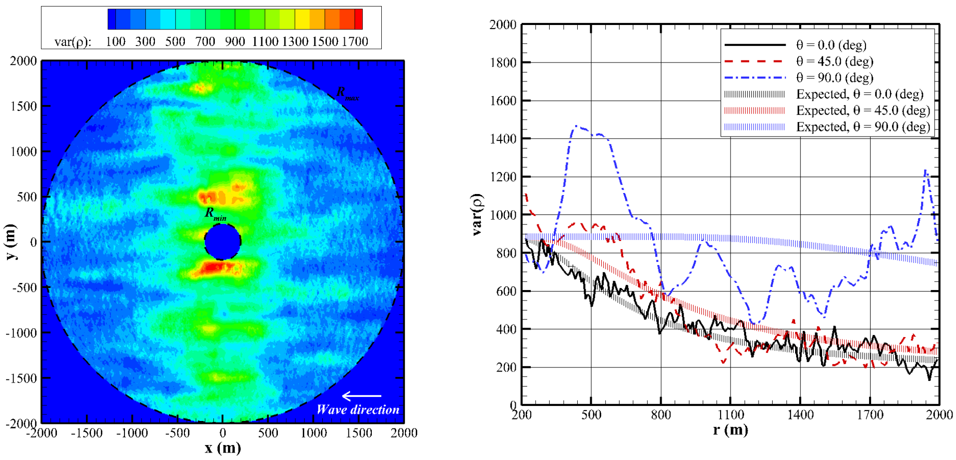


(a) Fitting with Smith function (b) Estimated mean surface slope

Fig. 3 Estimation of the mean surface slope by fitting with Smith function

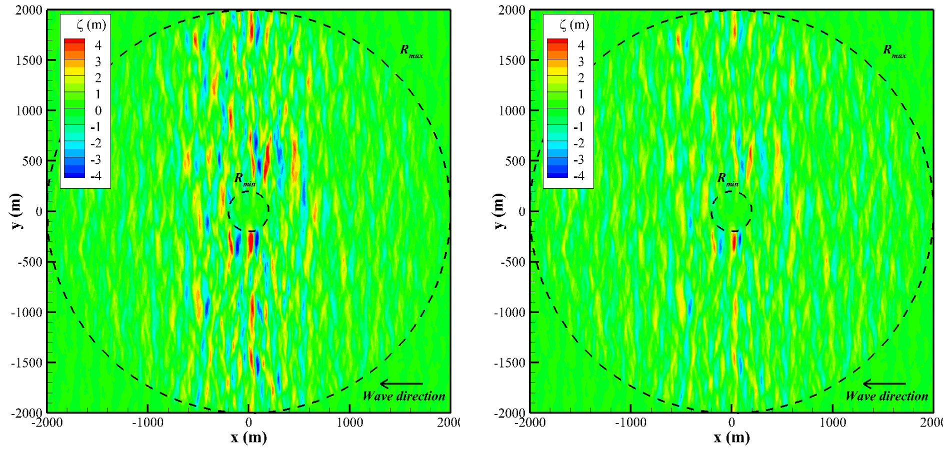
Fig. 4 presents the observed variance of radar image intensities. It can be confirmed that the variance is non-uniform because of the dependence of shadowing effects on the azimuth; the shadowing effects are severe for the main propagation direction and weak in cross-wave direction. The expected variance is evaluated with the mean surface slope according to Eq. (5), and compared with the observed variance for each azimuthal direction. The estimated and observed variances are in good agreement as shown in Fig. 4(b).

Lastly, the radar image intensity is calibrated with the theoretically derived variance. Subsequently, the general wave-field reconstruction is conducted based on the 3D FFT & IFFT analysis techniques. Fig. 5 shows the reconstructed results with or without the energy-level calibration. As shown in Fig. 5(a), the reconstructed wave elevation is overestimated in the cross-wave direction due to the nonhomogeneous energy distribution. On the other hand, mitigation of the overestimation by the calibration is confirmed (refer to Fig. 5(b)). When the results are assessed through the root-mean-square (RMS) error between the exact and reconstructed wave fields, the accuracy is enhanced by about 10% for the whole measurement domain by the calibration.



(a) Variance distribution (b) Variance in each direction

Fig. 4 Variance of radar image intensity: Short-crested waves of Beaufort scale 7 (H_S=4.0 m, T_{mean} = 7.7 s)



(a) w/o calibration (b) w/ calibration

Fig. 5 Reconstructed wave-field: Short-crested waves of Beaufort scale 7 (H_S=4.0 m, T_{mean} = 7.7 s)

4 CONCLUSIONS

The following conclusions can be made throughout this study:

- The accuracy of the significant wave height estimation is improved by considering the spatial autocorrelation effects in the shadowing function.
- The non-uniformity of the reconstructed wave field can be mitigated by applying the energy-level calibration based on the theoretically derived variance of radar image intensities.

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