Wave power extraction from a metamaterial cylinder with internal paddle power take off mechanism

J. Huang, R. Porter
School of Mathematics, University of Bristol, Bristol BS8 1UG, UK
jin.huang@bristol.ac.uk, richard.porter@bristol.ac.uk

1 INTRODUCTION

Originating in the field of electromagnetics, the term metamaterial is used to describe a microstructure which interacts with the wavefield to create unusual effects not normally associated with scattering in ordinary settings. The microstructure is usually formed by a periodic arrangement of elements whose design contributes to the macroscopic properties of the metamaterial. In water waves, one such metamaterial device used recently by a number of authors is a closely-spaced periodic array of thin vertical bottom-mounted plates. For example, it has been shown in [1], [2] that submerged rectangular ridges are capable of perfectly transmitting negatively-refracted plane incident waves. When plate arrays are formed into vertical circular cylinders extending through the depth [3] have demonstrated that there is capacity for the cylinder to absorb significantly more wave energy than an equivalent rigid cylinder operating in rigid body motion, exploiting the internal resonance associated with the metamaterial properties of the plate array microstructure. [3] considered placing a damping condition on the internal free surface of the cylinder and imagined how this could be representative of a physical mechanical absorbing mechanism. Meanwhile, in a separate study of [4] in which narrow vertical absorbing paddles were placed around the surface of a vertical cylinder also showed the ability of a circular cylinder to absorb more energy than in rigid body motion.

In this paper, we return to [3] and consider a more realistic way to extract the power from the internal resonance promoted by the metamaterial structure. We arrange a pair of hinged paddles with springs and dampers about the midplane of each channel. The paddles are excited by water waves to rotate around the hinge points and the dampers are used to control the power absorption. To simplify the mathematical problem, an assumption is made in which the discrete paddle displacement in discrete channels is replaced by a continuous double-sided paddle displacement along the centreplane of a cylinder occupied with a continuous effective medium. The results show that this new type of wave energy converter (WEC) can capture significantly larger energy in almost all incident wave frequencies and angles when compared to the traditional WECs operating in rigid body motion.

2 PROBLEM FORMULATION

As shown in Fig.1, a parallel array of closely-spaced vertical thin plates aligned with y-axis are confined within a circular cylinder of radius a centred at the origin in the water of depth h and density ρ. Between adjacent plates, there are a pair of paddles hinged at $z = -c$ with the identical damping, γ, and spring, κ. The paddles are fitted above rigid walls occupying $-h < z < -c$. The paddles are perpendicular to the plates and symmetrically distributed on either side of the symmetry plane of the cylinder. Power is generated under the action of a regular wave with the amplitude A which propagates at an angle β to the positive x-axis. It should be noted that there is no fluid between the two rows of paddles and the separation between these two rows must be large enough that the paddles do not come into contact in operation.

The whole fluid domain is divided into two regions: an outer region $\Omega_1 = \{r > a, 0 \le \theta < 2\pi, -h \le z \le 0\}$ and an inner region $\Omega_2 = \{r \le a, 0 \le \theta < 2\pi, -h \le z \le 0\}$, where (r, θ) is the polar coordinate.

Linear potential theory is used to study the present problem and in which the fluid motion is described by a velocity potential $\Re[\phi(x, y, z)\mathrm{e}^{\mathrm{i}\omega t}]$, where ω is the assumed angular frequency. Therefore,

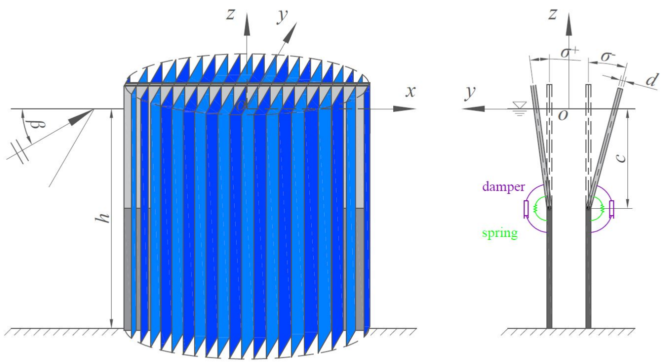

Figure 1: Sketch for the metamaterial cylinder with paddles.

the time-independent velocity potentials $\phi_k(x,y,z)$ satisfy

$$\nabla^2\phi_k(x,y,z)=0 \quad \text{in} \quad \Omega_k \quad (k=1,2). \tag{1}$$

with

$$\frac{\partial\phi_k}{\partial z}=\frac{\omega^2}{g}\phi_k \quad \text{on} \quad z=0, \quad \text{and} \quad \frac{\partial\phi_k}{\partial z}=0 \quad \text{on} \quad z=-h. \tag{2}$$

In the inner region, the velocity potential also satisfies no-flow conditions on either side of each vertical channel wall. On the underlying assumption that the the width of the channel is much smaller its length and water depth a multiscale homogenisation (e.g. Porter (2021)) can be used to replace the combined internal fluid/structure by an effective medium governing equation

$$\frac{\partial^2\phi_2}{\partial y^2}+\frac{\partial^2\phi_2}{\partial z^2}=0 \quad \text{in} \quad \Omega_2. \tag{3}$$

The velocity potential $\phi_2(x,y,z)$ also satisfies kinematic and dynamic boundary conditions on the surface of paddles. Since the width of the paddles is assumed small and the number of the paddles consequently large, we assume that the pitch angles of the two paddle rows can be expressed as continuous functions, $\sigma^{\pm}(x)$, of position x. For modelling simplicity, the distance of between the two row of paddles and the thickness of the paddles (assumed much smaller than the radius of the cylinder), are set to zero in the present study. Thus, the time-harmonic kinematic condition on the two rows of paddles at $y=0^{\pm}$, $-a<x<a$ is written

$$\frac{\partial\phi_2(x,y,z)}{\partial y}=\begin{cases} -\mathrm{i}\omega\sigma^{\pm}(x)(z+c), & -c<z<0 \\ 0, & -h<z<-c \end{cases} \tag{4}$$

and the dynamic condition expressed as

$$(-\omega^2 I_c+\mathrm{i}\omega\gamma+\kappa+C)\sigma^{\pm}(x)=\mathrm{i}\omega\rho\int_{-c}^{0}\phi_2(x,0^{\pm},z)(z+c)\,\mathrm{d}z, \qquad -a<x<a \tag{5}$$

where $I_c=\frac{1}{3}\rho_s cd\left(c^2+\frac{1}{2}d^2\right)$ and $C=\frac{1}{2}\rho gd(\frac{1}{6}d^2+c^2)-\frac{1}{2}\rho_s gc^2 d$ are the moment inertia and restoring moment per unit width for the paddles hinged along the center of the bottom edge. Here ρ_s represents the density the paddle.

Applying the continuity of pressure and flux on the interface of the inner and outer regions, we can obtain the corresponding matching conditions on the interface

$$\phi_1=\phi_2, \quad \text{and} \quad \frac{\partial\phi_1}{\partial r}=\frac{\partial\phi_2}{\partial y}\sin\theta, \quad \text{on} \quad r=a. \tag{6}$$

Since the structure possesses geometric symmetry about $y=0$, the total potential can be written as the sum of potential which are symmetric and anti-symmetric about $y=0$, i.e.

$$\phi_k=(\phi_k^s+\phi_k^a)/2, \quad (k=1,2), \quad \text{where} \quad \begin{cases} \phi_k^s(x,-y,z)=\phi_k^s(x,y,z), \\ \phi_k^a(x,-y,z)=-\phi_k^a(x,y,z). \end{cases} \tag{7}$$

Consequently, we only need to consider solutions in the upper half domain ($y \geq 0$) provided we apply appropriate conditions on $y = 0$ and introduce functions $\sigma^{s,a}(x)$ from which $\sigma^{\pm}(x) = \frac{1}{2}(\sigma^s(x) \pm \sigma^a(x))$.

In the outer region, the solutions for the symmetric potential ϕ_1^s satisfying Eqs. (1), (2) and (7) can be written as

$$\phi_1^s = -\frac{\mathrm{i}gA}{\omega} \sum_{m=0}^{\infty} \cos m\theta \left[2\epsilon_m \mathrm{i}^m \cos m\beta J_m(kr)\psi_0(z) + A_{m0}^s H_m(kr)\psi_0(z) + \sum_{n=1}^{\infty} A_{mn}^s K_m(k_n r)\psi_n(z) \right], \tag{8}$$

where the first term in the bracket represents the (symmetric) incident wave and the second and third terms represent the scattered response. Here, $\epsilon_0 = 1$ and $\epsilon_m = 2$, ($m \geq 1$), k_n ($n = 1, 2, \dots$) are the real positive roots of the dispersion equation $\omega^2 = -gk_n \tan k_n h$ while k is the wave number defined by $k_0 = -\mathrm{i}k$, $\psi_n(z)$ are the vertical eigenfunctions defined by $\psi_n(z) = \cos k_n(z+h)/\cos k_n h$ and A_{mn}^s are the undetermined coefficients.

By applying the method of separation of variables, the symmetric velocity potential ϕ_2^s in the inner region satisfying Eqs. (2) and (3) can be written as

$$\phi_2^s = -\frac{\mathrm{i}gA}{\omega} \sum_{n=0}^{\infty} \left[C_n^s(x)\mathrm{e}^{-k_n y} + D_n^s(x)\mathrm{e}^{k_n y} \right] \psi_n(z). \tag{9}$$

To solve the resulting problem, it is convenient to expand the unknown functions $C_n^s(x)$ and $D_n^s(x)$ in Eq. (9) in a series of Chebyshev polynomials as $[C,D]_n^s(x) = \sum_{p=0}^{\infty} [C,D]_{n,p}^s T_p(x/a)$ where $C_{n,p}^s$ and $D_{n,p}^s$ are unknown coefficients to be determined.

The expressions for the anti-symmetric potentials ϕ_k^a in the inner and outer regions can be obtained after replacing cos functions with sin functions and replacing the superscript s with a.

By applying the symmetric/antisymmetric versions of the boundary conditions (4), (5), (6) to the expressions given above, and using the orthogonality of trigonometric functions, and vertical eigenfunctions $\psi_n(z)$, and after truncating the infinite series that result to finite series, a closed system of equations with unknowns $A_{mn}^{s/a}$, $C_{np}^{s/a}$ and $D_{np}^{s/a}$ can be formulated. Once these coefficients are determined, the wave elevation in the wave field can be calculated using the linearised kinematic condition on the free surface ($\eta = \frac{\mathrm{i}\omega}{g}\phi|_{z=0}$). The mean power absorption can be evaluated by integrating the time-averaged product of pressure and velocity over the paddle surfaces and this results in

$$P = \frac{\rho\omega}{4} \Im \left\{ \int_{-a}^{a} \int_{-c}^{0} \left[\phi_2^s(x, 0^+, z) \frac{\partial \phi_2^{s*}(x, 0^+, z)}{\partial y} + \phi_2^a(x, 0^+, z) \frac{\partial \phi_2^{a*}(x, 0^+, z)}{\partial y} \right] \mathrm{d}z \, \mathrm{d}x \right\} \tag{10}$$

where $*$ represents the complex conjugate. In order to represent the results in a meaningful way, the non-dimensional power $\bar{P}$ is defined as $\bar{P} = kP/P_i$ where $P_i = \frac{1}{2}\rho g A^2 c_g$ represents the incident wave power per unit width of the wave front and $c_g = (\omega/2k)(1 + 2kh/\sinh 2kh)$ is the group velocity.

3 RESULTS

We consider a metamaterial cylinder of radius a is in the water of depth $h = 2a$ at the incident wave angle $\beta = \pi/4$ with the paddles attached to a constant non-dimensional damping $\bar{\kappa} = \kappa/(\rho a c^2 \sqrt{gh}) = 0.2$ and spring $\bar{\gamma} = \gamma/(\rho g a c^2) = 0.2$. We consider four cases with the same paddle thickness $d = 0.1a$ and density $\rho_s = 2\rho$ but different paddle lengths, c. Fig. 2(a) shows the non-dimensional power $\bar{P}$ generated by the paddles against the incident wave frequency. It shows that the paddle length has little influence on the power extraction. The maximum mean power generated by an equivalent axisymmetric WEC operating in combined rigid-body heave and surge/pitch motion under this non-dimensionalisation is $\bar{P} = 3/2$ ([5]). For the present device, it can be found that $\bar{P} > 2$ for $ka > 1$. Fig. 2(b) and (c) show the amplitudes of the paddle motion at $ka = 1$. Although the paddle length is different, the horizontal displacements of the paddles at the mean surface are of the same order of

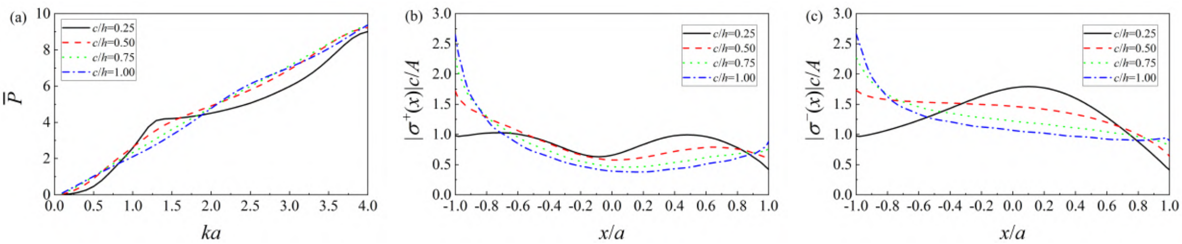

Figure 2: In (a) non-dimensional power generated by the metamaterial cylinder of radius a in water of depth $h = 2a$ for an incident wave angle $\beta = \pi/4$ with dimensionless damping $\bar{\gamma} = 0.2$ and spring $\bar{\kappa} = 0.2$. In (b) and (c) the corresponding amplitudes of the paddle motion at $ka = 1$.

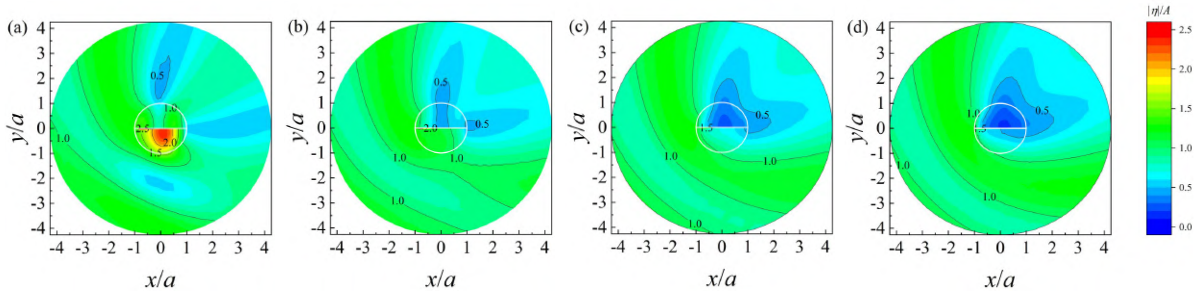

Figure 3: The wave elevation $h = 2a$ at $ka = 1$ and $\beta = \pi/4$ with the damping $\bar{\gamma} = 0.2$ and spring $\bar{\kappa} = 0.2$: (a) $c/h = 0.25$; (b) $c/h = 0.50$; (c) $c/h = 0.75$; (d) $c/h = 1.00$.

magnitude as the incident wave amplitude. Upside and leeside of the cylinder, the paddle tends to moves more with greater amplitude as paddle length increases.

The corresponding wave elevations at $ka = 1$ are presented in Fig. 3. Although the horizontal displacements of the paddles are similar to incident wave heights, the wave elevation – especially inside the metamaterial cylinder – are quite different. With the longer paddle length, the wave amplitudes induced by the paddles are relatively small. Remarkably, when the hinge point is close to the sea bed, the wave elevations on surfaces of the paddles become relatively calm.

Fig. 4 shows a contour plot of the dimensional wave power $\bar{P}$ generated by the metamaterial cylinder with the paddle length $c/h = 1.0$ against the non-dimensional frequency ka and incident wave angle β (due to symmetry, we need only consider $0 \leq \beta \leq \pi/2$).

The plot shows that, for most $ka \lesssim 3$, the power is not sensitive to the incident wave angle implying that this type of WEC can absorb high power from all wave directions.

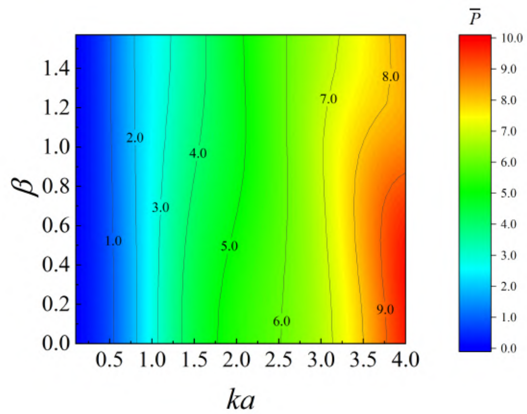

Figure 4: The contour of dimensional wave power generated by the metamaterial cylinder with the paddle length $c/h = 1$ against the non-dimensional frequency ka and incident wave angle β.

Acknowledgements: This work was supported by the EPSRC, grant number EP/V04740X/1

REFERENCES

[1] Porter, R. 2021. *Plate arrays as a perfectly-transmitting negative refraction metamaterial. Wave Motion 100*, 102673.

[2] Marangos, C., and Porter, R. 2021. *Shallow water theory for structured bathymetry. Proc. Roy. Soc. A. 477*, 102724.

[3] Zheng, S., Porter, R., and Greaves, D. 2020. *Wave scattering by an array of metamaterial cylinders. J. Fluid Mech. 903*, A50.

[4] Porter, R., Zheng, S., and Greaves, D. 2021. *Extending limits for wave power absorption by axisymmetric devices. J. Fluid Mech. 924*, A39.

[5] Evans, D. V. 1976. *A theory for wave power absorption by oscillating bodies. J. Fluid Mech. 77*, 1–25.