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Speed-effect restoring forces on ship motions

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Following a new decomposition of radiation forces into the sum of added mass, damping and restoring components, it is shown that the speed-effect restoring forces are critically important not only to keep the theoretical consistency but also to predict ship motions with much improved accuracy.

1 Unsteady potential of time-harmonic flow

We define a Cartesian coordinate system translating at the speed U with the ship in the positive x-direction. The z-axis is positive upwards with the origin at the undisturbed free surface. Relative to this reference frame, there exists an ambient flow $-U\boldsymbol{i}$ opposing the ship forward direction. The presence of ship in this ambient flow creates a ship-shaped steady flow around the hull, called base flow $\mathbf{W} = U\nabla(\bar{\phi}-x)$. In addition to this base flow, there should be a wavy steady flow $\nabla\phi$. When the ship oscillates about the reference frame or/and in incoming waves, there exists also an unsteady flow $\nabla\psi$. The wavy steady and unsteady flows are called perturbation flow and represented by the velocity potential $\Phi = \phi + \psi$. All velocity potentials $(\bar{\phi}, \phi, \psi)$ satisfy the Laplace equation in the fluid. The total flow $\mathbf{W} + \nabla\Phi$ satisfies the kinematic and dynamic conditions written in the combined form

$$\begin{aligned} &\Phi_{tt} + g\Phi_z + 2\mathbf{W}\cdot\nabla\Phi_t + \mathbf{W}\cdot\nabla(\mathbf{W}\cdot\nabla\Phi) + \nabla\Phi\cdot(\mathbf{W}\cdot\nabla)\mathbf{W} \\ &= -2\nabla\Phi\cdot\nabla\Phi_t - (\mathbf{W}+\nabla\Phi)\cdot(\nabla\Phi\cdot\nabla)\Phi - \nabla(\mathbf{W}\cdot\nabla\Phi)\cdot\nabla\Phi - gU\bar{\phi}_z - \mathbf{W}\cdot(\mathbf{W}\cdot\nabla)\mathbf{W} \end{aligned} \tag{1}$$

on the free surface $z = \eta$ which is defined by

$$\eta = -\frac{1}{g}\Big[(\partial_t + \mathbf{W}\cdot\nabla)\Phi + \frac{1}{2}\nabla\Phi\cdot\nabla\Phi + \frac{1}{2}(\mathbf{W}\cdot\mathbf{W} - U^2)\Big] \tag{2}$$

The above equations (1) for potentials $\Phi = \phi + \psi$ and (2) for wave elevations are fully nonlinear with quadratic and cubic products of potentials and the assumption of time independence concerns only the base flow $\mathbf{W}$. Obtaining solutions of such problems with accuracy is extremely difficult if not impossible.

We assume now the base flow $\mathbf{W} = U\nabla(\bar{\phi} - x)$ is of order $O(1)$ while the perturbation flows $\Phi = \phi + \psi$ are of smaller order $o(1)$ comparing to the base flow $(\bar{\phi} - x)$ in the same way presented in Sclavounos (1995). Thus, the quadratic and cubic products of (ϕ, ψ) on the right-hand side of (1) are ignored. Furthermore, the free-surface elevation η is also assumed to be of smaller order $o(1)$ which is true for small or moderate speed. The Taylor expansion of all terms in (1) with respect to $z = 0$ can be used to express the boundary condition on the mean free surface. Finally, the frequency-domain expression of unsteady potential is scaled and written as

$$\psi = \Re\mathrm{e}\{\varphi\, \mathrm{e}^{-\mathrm{i}\omega_e t}\} L\sqrt{gL} \tag{3}$$

with ω_e the encounter frequency, g the acceleration due to gravity and L the ship length. The base flow $\mathbf{W} = U\mathbf{w}$ with $\mathbf{w} = \nabla\bar{\phi} - \boldsymbol{i}$ is used to obtain the linear boundary condition for the unsteady flow φ

$$\varphi_z - \omega^2\varphi - 2\mathrm{i}\tau\mathbf{w}\cdot\nabla\varphi + F_\mathrm{r}^2\mathbf{w}\cdot\nabla(\mathbf{w}\cdot\nabla\varphi) + F_\mathrm{r}^2\nabla\varphi\cdot(\mathbf{w}\cdot\nabla)\mathbf{w} + \bar{\phi}_{zz}(\mathrm{i}\tau\varphi - F_\mathrm{r}^2\mathbf{w}\cdot\nabla\varphi) = 0 \tag{4}$$

on the mean free surface $z = 0$. In (4), we have used the notations $\omega = \omega_e\sqrt{L/g}$ for dimensionless encounter frequency, $F_\mathrm{r} = U/\sqrt{gL}$ the Froude number and $\tau = \omega F_r$ the Brard number. Without considering the presence of ship, the classical Neumann-Kelvin linearisation can be derived by taking $\bar{\phi} = 0$ (then $\mathbf{w} = -\boldsymbol{i}$) in (4)

$$\varphi_z - \omega^2\varphi + 2\mathrm{i}\tau\varphi_x + F_\mathrm{r}^2\varphi_{xx} = 0 \tag{5}$$

which is widely used in many previous studies. The Neumann-Kelvin boundary condition (5) can be illustrated as the uniform stream penetrating in and through the ship hull (physically unacceptable) as shown by the left part of Figure 1. The linearisation based on the ship-shaped stream (4) is illustrated on the right part of Figure 1 which is physically acceptable.

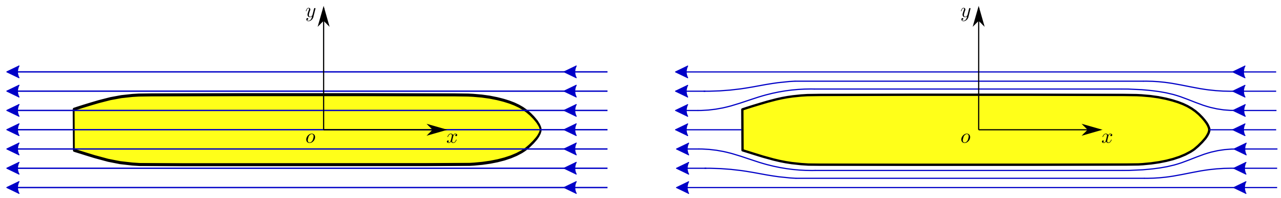

Figure 1: Uniform stream (left) *vs* ship-shaped stream (right) as the base flow in the linearisation

The boundary condition on the ship hull is written on H at its mean position

$$\frac{\partial}{\partial n}\varphi = \begin{cases} -\varphi_n^I & \text{diffraction} \\ \sum_{j=1}^{6}\left(-\mathrm{i}\omega\xi_j n_j + F_\mathrm{r}\xi_j m_j\right) & \text{radiations} \end{cases} \tag{6}$$

The potential φ^I representing incoming waves is well known. The six elementary motions are denoted by ξ_j for $j = 1, 2, \cdots, 6$ including the translations $\boldsymbol{T} = (\xi_1, \xi_2, \xi_3)$ and rotations $\boldsymbol{R} = (\xi_4, \xi_5, \xi_6)$. The vector components $\{n_j\}$ for $j = 1, 2, \cdots, 6$ are elements of the generalized normal vector $\{\boldsymbol{n}, \boldsymbol{r}\wedge\boldsymbol{n}\}$ defined in (13) and $\{m_j\}$ for $j = 1, 2, \cdots, 6$ are those of m_j terms

$$(m_1, m_2, m_3) = -(\boldsymbol{n}\cdot\nabla)\mathbf{w} \quad \text{and} \quad (m_4, m_5, m_6) = -(\boldsymbol{n}\cdot\nabla)(\boldsymbol{r}\wedge\mathbf{w}) \tag{7}$$

given in Newman (1978), depending on the ship-shaped stream $\mathbf{w}$ and the position vector $\boldsymbol{r}$. The normal vector $\boldsymbol{n} = (n_1, n_2, n_3)$ is defined positive inwards to the fluid.

This linearised boundary value problem with the condition (4) on the mean free surface and the condition (6) on the ship hull has been solved in Chen et al. (2021) by using the Green function associated with a pulsating and translating source with the viscous effect, and developing its analytical integration on the hull and on the free surface. The boundary integral equation includes that on the hull, that on the zone of free surface in the vicinity of ship and one over the waterplane inside the ship to ensure the good conditioned system of solutions. Numerical results in excellent agreement with experimental measurements shows that the method in Chen et al. (2021) provides well a reliable and practical method to evaluate wave loading and induced ship motions. One of important features in this new Green function method concerning a new decomposition of radiation forces is presented here.

2 Decomposition of radiation forces

According to the boundary condition (6) on the ship hull H, the time-harmonic potential φ can be written as the sum $\varphi = \varphi^D + \varphi^R$ of the diffraction φ^D and radiation φ^R. The radiation potential φ^R is further decomposed as

$$\varphi^R = -\mathrm{i}\omega\sum_{j=1}^{6}\xi_j\varphi_j^n + F_\mathrm{r}\sum_{j=1}^{6}\xi_j\varphi_j^m \tag{8}$$

in which the first 6 elementary potentials are due to ship velocities $-\mathrm{i}\omega\xi_j$ for $j = 1, 2, \cdots, 6$ and the second 6 potentials associated with ship's motions interacting with the base flow $\mathbf{w}$ via m_j terms. The boundary conditions on the hull for $(\varphi_j^n, \varphi_j^m)$ are then written by

$$\frac{\partial}{\partial n}\varphi_j^n = n_j \quad \text{and} \quad \frac{\partial}{\partial n}\varphi_j^m = m_j \tag{9}$$

for $j = 1, 2, \cdots, 6$. The time-harmonic pressure scaled with $(\rho g L)$ can be obtained by Bernoulli's equation

$$p^R = -\Big[-\mathrm{i}\omega\varphi^R + F_\mathrm{r}(\mathbf{w}\cdot\nabla\varphi^R)\Big] = \sum_{j=1}^{6}\xi_j\Big[\omega^2\varphi_j^n + \mathrm{i}\tau\big(\mathbf{w}\cdot\nabla\varphi_j^n + \varphi_j^m\big) - F_\mathrm{r}^2\mathbf{w}\cdot\nabla\varphi_j^m\Big] \tag{10}$$

by introducing the radiation potential (8) and considering the notation for the time derivative of the time-harmonic potential $\psi_t = -\mathrm{i}\omega\varphi$. In addition, the hydrostatic pressure p^H and that p^S due to the base flow are written by

$$p_0 = p^H + F_\mathrm{r}^2 p^S \quad \text{with} \quad p^H = -z \quad \text{and} \quad p^S = -\frac{1}{2}(\mathbf{w}\cdot\mathbf{w} - 1) = -\frac{1}{2}\big(\nabla\bar{\phi}\cdot\nabla\bar{\phi} - 2\bar{\phi}_x\big) \tag{11}$$

which yield the usual hydrostatic stiffness due to the spatial variations of p^H and of the ship hull associated with the ship motion, and the speed-effect restoring forces due to the variations linked to the base-flow pressure p^S.

The radiation forces including linear forces scaled by $(\rho g L^3)$ and torque moments scaled by $(\rho g L^4)$, are defined as the reaction of fluid via pressure on the ship hull against the acceleration, velocity and displacement of the ship, under the chosen reference system associated with the mean position of the ship

$$\boldsymbol{F}^R = -\iint_H p^R\boldsymbol{N}\,\mathrm{d}s - \iint_H \big(\boldsymbol{X}\cdot\nabla p_0 + p_0\boldsymbol{R}\wedge\big)\boldsymbol{N}\,\mathrm{d}s - \oint_\Gamma(\eta^R - X_3)p_0\boldsymbol{N}\,\mathrm{d}l \tag{12}$$

for $i = 1, 2, \cdots, 6$. In (12), $\boldsymbol{N}$ is called generalized normal vector, and $\boldsymbol{X}$ the displacement vector

$$\begin{aligned} &\boldsymbol{N} = \{\boldsymbol{n}, \boldsymbol{r} \wedge \boldsymbol{n}\} = \{n_1, n_2, n_3, n_4, n_5, n_6\} \\ &\boldsymbol{X} = \boldsymbol{T} + \boldsymbol{R} \wedge \boldsymbol{r} = \{X_1, X_2, X_3\} \qquad \text{with} \quad \boldsymbol{r} = \{x - x_0, y - y_0, z - z_0\} \end{aligned} \tag{13}$$

associated with the translation/rotation ($\boldsymbol{T}/\boldsymbol{R}$) and the position vector $\boldsymbol{r}$ with respect to the reference point $\boldsymbol{r}_0 = (x_0, y_0, z_0)$. The notation $\boldsymbol{R} \wedge \boldsymbol{N} = \{\boldsymbol{R} \wedge \boldsymbol{n}, \boldsymbol{R} \wedge (\boldsymbol{r} \wedge \boldsymbol{n})\}$ is adopted. The waterline integral is resultant from the integration on the intermittant surface between the local free-surface elevation $\eta^R = -\mathrm{i}\omega\varphi^R + F_\mathrm{r}(\mathbf{w} \cdot \nabla\varphi^R)$ and the vertical displacement of the waterline. This term often omitted in many studies is kept in Chen (2021). In addition to the projection of gravity force on the moving ship, we write

$$\boldsymbol{F}^R + \big\{\boldsymbol{0}, (\boldsymbol{R} \wedge \boldsymbol{r}_G) \wedge \boldsymbol{G}\big\} = -\sum_{j=1}^{6} \xi_j \big(-\omega^2 A_{ij} - \mathrm{i}\omega B_{ij} + C_{ij} \big) \tag{14}$$

in which A_{ij}, B_{ij} and C_{ij} on the right-hand side are called added-mass, damping and stiffness coefficients, respectively, following their respective association with the acceleration, velocity and displacement of the ship. The gravity $\boldsymbol{G}$ scaled by $(\rho g L^3)$ is equal to $(0, 0, -V/L^3)$ with V the buoyant volume in still water and $\boldsymbol{r}_G$ the center of gravity.

Among all terms in (12) and (14), the simplest ones are associated with the hydrostatic pressure $p^H = -z$

$$-\iint_H \big(\boldsymbol{X} \cdot \nabla p^H + p^H \boldsymbol{R} \wedge \big) \boldsymbol{N} \, \mathrm{d}s + \big\{\boldsymbol{0}, (\boldsymbol{R} \wedge \boldsymbol{r}_G) \wedge \boldsymbol{G}\big\} = -\sum_{j=1}^{6} \xi_j H_{ij} \tag{15}$$

in which H_{ij} is called hydrostatic restoring coefficients given in all textbooks. In a similar way, the contribution by p^S defined in (11) can be written by

$$-F_\mathrm{r}^2 \iint_H \big(\boldsymbol{X} \cdot \nabla p^S + p^S \boldsymbol{R} \wedge \big) \boldsymbol{N} \, \mathrm{d}s + F_\mathrm{r}^2 \oint_\Gamma X_3 p^S \boldsymbol{N} \, \mathrm{d}l = -F_\mathrm{r}^2 \sum_{j=1}^{6} \xi_j S_{ij} \tag{16}$$

where S_{ij} is called speed-effect restoring coefficients. Concerning the radiation forces due to the time-harmonic pressure p^R defined by (10) and the free-surface elevation η^R along the waterline, we decompose them

$$-\iint_H p^R \boldsymbol{N} \, \mathrm{d}s - F_\mathrm{r}^2 \oint_\Gamma \eta^R p_0^S \boldsymbol{N} \, \mathrm{d}l = -\sum_{j=1}^{6} \xi_j \Big(-\omega^2 R_{ij}^\omega - \mathrm{i}\tau R_{ij}^\tau + F_\mathrm{r}^2 R_{ij}^{F_\mathrm{r}} \Big) \tag{17}$$

with the components

$$\begin{cases} R_{ij}^\omega = -\displaystyle\iint_H \varphi_j^n n_i \, \mathrm{d}s + F_\mathrm{r}^2 \oint_\Gamma \varphi_j^n p_0^S n_i \, \mathrm{d}l \\ R_{ij}^\tau = -\displaystyle\iint_H \big(\mathbf{w} \cdot \nabla\varphi_j^n + \varphi_j^m\big) n_i \, \mathrm{d}s + F_\mathrm{r}^2 \oint_\Gamma \big(\mathbf{w} \cdot \nabla\varphi_j^n + \varphi_j^m\big) p_0^S n_i \, \mathrm{d}l \\ R_{ij}^{F_\mathrm{r}} = -\displaystyle\iint_H \big(\mathbf{w} \cdot \nabla\varphi_j^m\big) n_i \, \mathrm{d}s + F_\mathrm{r}^2 \oint_\Gamma \big(\mathbf{w} \cdot \nabla\varphi_j^m\big) p_0^S n_i \, \mathrm{d}l \end{cases} \tag{18}$$

By introducing (15), (16) and (17) back to (12) and following the usual notations, the coefficients (A_{ij}, B_{ij}, C_{ij}) on the right-hand side of (14) are given by

$$\begin{cases} A_{ij} = \Re\mathrm{e}\Big\{R_{ij}^\omega + \mathrm{i}(F_\mathrm{r}/\omega) R_{ij}^\tau - (F_\mathrm{r}^2/\omega^2)\big(R_{ij}^{F_\mathrm{r}} - R_{ij}^S\big)\Big\} \\ B_{ij} = \Im\mathrm{m}\Big\{\omega R_{ij}^\omega + \mathrm{i} F_\mathrm{r} R_{ij}^\tau - (F_\mathrm{r}^2/\omega) R_{ij}^{F_\mathrm{r}}\Big\} \\ C_{ij} = H_{ij} + F_\mathrm{r}^2 \left(S_{ij} + R_{ij}^S\right) \end{cases} \tag{19}$$

The terms R_{ij}^S in above (19) are defined by the limit at zero frequency of the component $R_{ij}^{F_\mathrm{r}}$ given in (18)

$$R_{ij}^S = R_{ij}^{F_\mathrm{r}}(\omega \to 0) \tag{20}$$

which are purely real.

3 Speed-effect restoring forces and ship motions

The new decomposition (14) of radiation forces (plus gravity contribution) into added-mass (A_{ij}), damping (B_{ij}) and restoring coefficients (C_{ij}) given by (19) includes two speed-effect restoring coefficients S_{ij} contributed by the steady pressure (16) and R_{ij}^S by the "unsteady" pressure at the limit of zero frequency (20). The speed-effect restoring forces R_{ij}^S are subtracted from the added-mass coefficients so that A_{ij} in (19) tend to a *finite* value at $\omega \to 0$ unlike the classical added-mass coefficients which tend to infinity as $O(\omega^{-2})$ by ignoring the speed-effect restoring forces.

In addition to the classical hydrostatic restoring coefficients H_{ij} defined by (15), the total restoring coefficients C_{ij} in (19) are augmented by the speed-effect components $(S_{ij} + R_{ij}^S)$ multiplied by the factor F_r^2. These additional components are then negligible for $F_\mathrm{r} \to 0$ as expected. The component R_{ij}^S can be approximated by

$$R_{ij}^S = -\iint_H n_i(\mathbf{w} \cdot \nabla \bar{\varphi}_j^m)\, \mathrm{d}s + O(F_\mathrm{r}^2) \tag{21}$$

by ignoring the waterline integral of order $O(F_\mathrm{r}^2)$ on the right-hand side of (18). The zero-frequency "unsteady" potentials $\bar{\varphi}_j^m$ associated with the m_j terms can be approximated by

$$\{\bar{\varphi}_j^m\}_{j=1,2,\cdots,6} = -\{\mathbf{w}, \boldsymbol{r} \wedge \mathbf{w}\} + O(F_\mathrm{r}^2) \tag{22}$$

since they satisfy

$$\frac{\partial}{\partial n}\bar{\varphi}_j^m = \boldsymbol{n} \cdot \nabla \bar{\varphi}_j^m = m_j \quad \text{and} \quad \frac{\partial}{\partial z}\bar{\varphi}_j^m = 0 \tag{23}$$

on the hull for $j = 1, 2, \cdots, 6$ and on the mean free surface for $j = (1, 2, 6)$, respectively.

On the other side, the component S_{ij} defined by (16) is converted into another form in Chen (2021)

$$\{S_{ij}\}_{i=1,2,6} = \iint_H \{\nabla p^S, \boldsymbol{r} \wedge \nabla p^S\} n_j\, \mathrm{d}s \tag{24}$$

for $j = (1, 2, 6)$. Introducing (8) for $\bar{\varphi}_j^m$ in (21) and (11) for p^S in (24), we can show that

$$S_{jj} + R_{jj}^S = 0 \tag{25}$$

for $j = (1, 2, 6)$, i.e., in the horizontal directions (surge, sway and yaw). This cancellation of the speed-effect restoring forces in the horizontal directions is expected to be consistent with physics. On the contrary, the neglect of the component S_{ij} should induce non-physical restoring forces represented by R_{ij}^S and the accuracy of ship motions could be affected, in particular, in the horizontal directions where no other restoring forces are present.

Even in the vertical directions (heave, roll and pitch) where the hydrostatic restoring forces are dominant, the influence of speed-effect restoring forces S_{ij} is significant. Indeed, the speed-effect restoring coefficients S_{ij} are computed based on the ship-shaped stream $\mathbf{w}$ and introduced in the added-mass coefficients (19) in place of $(-R_{ij}^S)$. The comparison of A_{55} is depicted on the left of Figure 2, and that of pitch RAO on the right of the figure. It is shown that the inclusion of S_{ij} is critically important to predict ship motions with accuracy.

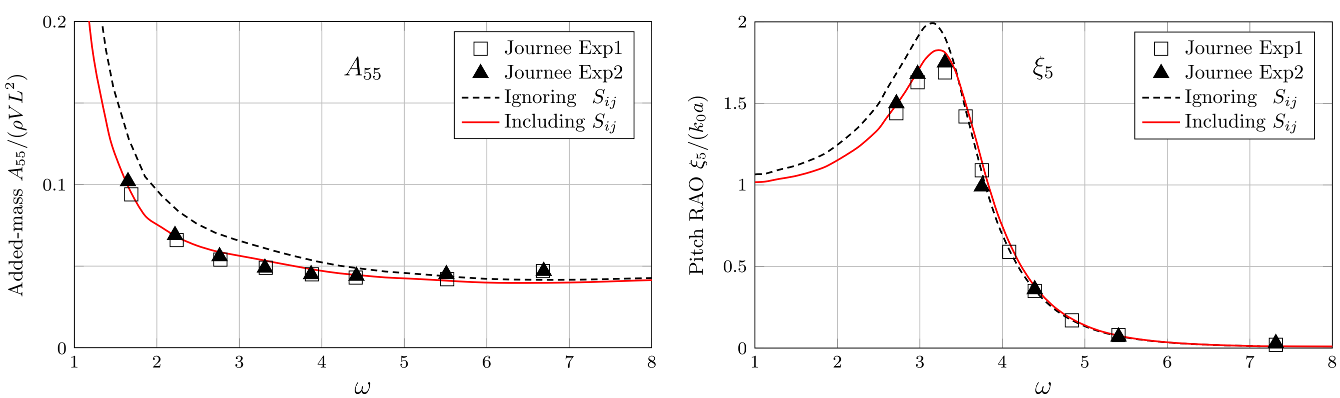

Figure 2: Added-mass coefficient (left) and Pith RAO (right) including (solid) the speed-effect restoring forces S_{ij} *versus* those ignoring S_{ij} (dashed), comparing with measurements of head-sea tests on Wigley IV hull at $F_\mathrm{r} = 0.3$ in Journee (1992).

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