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The Challenges of Computing Wave Added Resistance Using Maruo's Formulation and The Kochin Function

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1 Introduction

Maruo's formulation [1] for the wave drift force on bodies with forward speed (added resistance), requires calculation of finite and semi-infinite integrals containing the Kochin function. With this abstract we intend to highlight and discuss some computational challenges related to these integrals in further detail through several two- and three-dimensional (2D, 3D) example calculations.

2 The formulation

According to Maruo's formulation [1, 2] the added resistance R_w can be computed by

$$R_w = \frac{\rho}{4\pi}\left\{-\int_{-\infty}^{\bar{k}_1} + \int_{\bar{k}_2}^{\bar{k}_3} + \int_{\bar{k}_4}^{\infty}\right\} \frac{\bar{\kappa}\,(m - K\cos\beta)}{\sqrt{\bar{\kappa}^2 - m^2}} \left|H(m)\right|^2 dm, \tag{2.1}$$

in which ρ is the fluid density, $K = \omega_0^2/g$ is the wave number and β is the heading angle, with ω_0 the wave frequency, and g the gravitational acceleration. The integration bounds are

$$\bar{k}_1 = -\frac{\kappa_0}{2}\left(1 + 2\tau + \sqrt{1+4\tau}\right), \quad \bar{k}_3 = \frac{\kappa_0}{2}\left(1 - 2\tau - \sqrt{1-4\tau}\right),$$
$$\bar{k}_2 = -\frac{\kappa_0}{2}\left(1 + 2\tau - \sqrt{1+4\tau}\right), \quad \bar{k}_4 = \frac{\kappa_0}{2}\left(1 - 2\tau + \sqrt{1-4\tau}\right).$$

Here $\kappa_0 = g/U^2$ and $\tau = U\omega/g$, U as the forward speed of the body and ω as the encounter frequency. Note that if $\tau > \frac{1}{4}$ then $\bar{k}_3 = \bar{k}_4 = \kappa_0\tau$, and the last two integrals are merged in to one semi-infinite integral in $[\bar{k}_2\ \infty]$. The Kochin function is defined by an integration over the body surface $\boldsymbol{S}_b$

$$H(m) = \iint_{\boldsymbol{S}_b} \left(\phi_B \frac{\partial}{\partial n} - \frac{\partial \phi_B}{\partial n}\right) e^{\bar{\kappa}z + \mathrm{i}\left(xm + y\sqrt{\bar{\kappa}^2 - m^2}\right)} ds, \tag{2.2}$$

where $\partial/\partial n$ denotes the derivative normal to the body surface, ϕ_B is the combined radiation and scattering velocity potential and $\bar{\kappa}(m) = \frac{1}{g}\left(\omega + mU\right)^2$. In the context of the 2D strip theory, (2.2) can be expressed by

$$H(m) = \int_L e^{\mathrm{i}xm}\left[\int_{Cx} e^{\bar{\kappa}z} e^{\mathrm{i}y\sqrt{\bar{\kappa}^2 - m^2}} \left\{\left(\mathrm{i}\sqrt{\bar{\kappa}^2 - m^2} N_y + \bar{\kappa} N_z\right)\psi_B - \left(\frac{\partial \psi_B}{\partial N}\right)\right\} dl\right] dx, \tag{2.3}$$

where N denotes the normal vector to each 2D section C_x in the $y - z$ plane, and L is length of the body. The total 2D disturbance potential is ψ_B. Using the method of equivalent line source, Kashiwagi [2, 3] proposed also an equivalent equation for the Kochin function inside his *Enhanced Unified Theory*. In the 2D strip theory, this alternative equation takes the following form

$$H(m) = \int_L e^{\mathrm{i}xm}\left[\int_{C_x} e^{\nu z} e^{\mathrm{i}\nu y} \left\{\left(\mathrm{i}\nu N_y + \nu N_z\right)\psi - \left(\frac{\partial \psi}{\partial N}\right)\right\} dl\right] dx, \tag{2.4}$$

in which ψ is either the radiation ($\nu = \omega^2/g$) or the scattering velocity potential ($\nu = K$).

3 Calculations and Discussions

We believe there are two unsolved issues related to the computation of wave added resistance using (2.1), and the 3D Kochin function (2.2) or its 2D equivalent (2.3). The first is evaluation of the semi-infinite integrals. For many years, in order to avoid the associated *computational difficulties*, these integrals have either been neglected completely (arguing that their contribution is negligible), or have been computed by assuming an *artificial* and *arbitrary* submergence of the z- coordinate of the floating body (asserting that it is necessary for *convergence*). Even if we assume that the added resistance can be calculated using only the middle integral in (2.1), then we are faced with a second issue which is the poor comparison between Maruo's far-field method and the well-established near-field method for *floating bodies*, (the best comparison yet achieved in our calculations is related to a floating spheroid [4]). In what follows we discuss these matters using 2D and 3D computations.

3.1 Alternative and Original form of the Kochin Function (2D)

To our best knowledge the semi-infinite integrals have never been treated completely, except by Prof. Kashiwagi through the alternative form of the Kochin function (2.4) and the Enhanced Unified Theory (see for example [2, 3]). In a recent paper [5] we have employed this alternative form inside our in-house strip-theory code which is an implementation of the classical STF strip theory using the low-order BEM and the zero-speed Green function. Note that Kashiwagi uses an elegant semi-analytical method to compute these integrals, but in our work we truncate the integration where there is almost zero contribution to the final results. Using these computations, we have illustrated the relative contributions of the semi-infinite integrals to the added resistance. An example is shown in figure 1. In the plot the subscripts on H and the superscripts on R_w denote the contribution to the corresponding integral in (2.1). The Froude number is given by Fr, B is the beam of the vessel, and A is the wave amplitude. As can be seen, the semi-infinite integrals play an important role in computing the added resistance, and should not be neglected. This fact has already been emphasized by Prof. Kashiwagi. It is also important to note that for $\tau > 1/4$ there is no finite integral in (2.1), and neglecting the third integral is in fact equivalent to cutting-off the semi-infinite integral at $\bar{k}_3 = \bar{k}_4$. In addition we have conducted a convergence study of the computations based on the original formulation (2.3) for the added resistance of the Wigley hull. See figure 2. The computation for added resistance of a prismatic barge is also shown in figure 3. In order to ensure accuracy in case of a highly oscillatory Kochin function for large m values, we employed a *spectral* integration scheme for the integral along x. According to these computations the convergence of the added resistance based on the original form (2.3) is achieved, however in neither of these cases are the converged results anything close to the correct solutions based on the alternative form of the Kochin function. This is also true if we consider only the middle integral in (2.1) for the added resistance, which has become common practice in the literature. For the Wigley hull the converged results are unrealistically large. In the next section we illustrate this issue further using 3D computations and only based on the original 3D form (2.2). The 3D

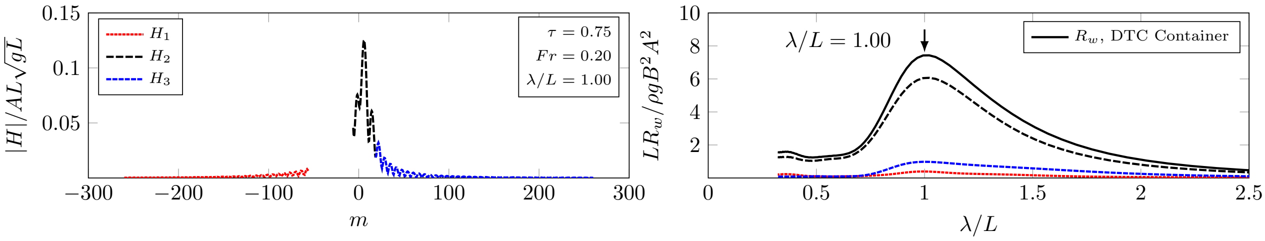

Figure 1: (Left) The Kochin function related to three integrals in (2.1). (Right) Illustration of relative contribution of the semi-infinite integrals to the added resistance of the DTC Container. Note the computations are based on the ***alternative*** form of the Kochin function (2.4)

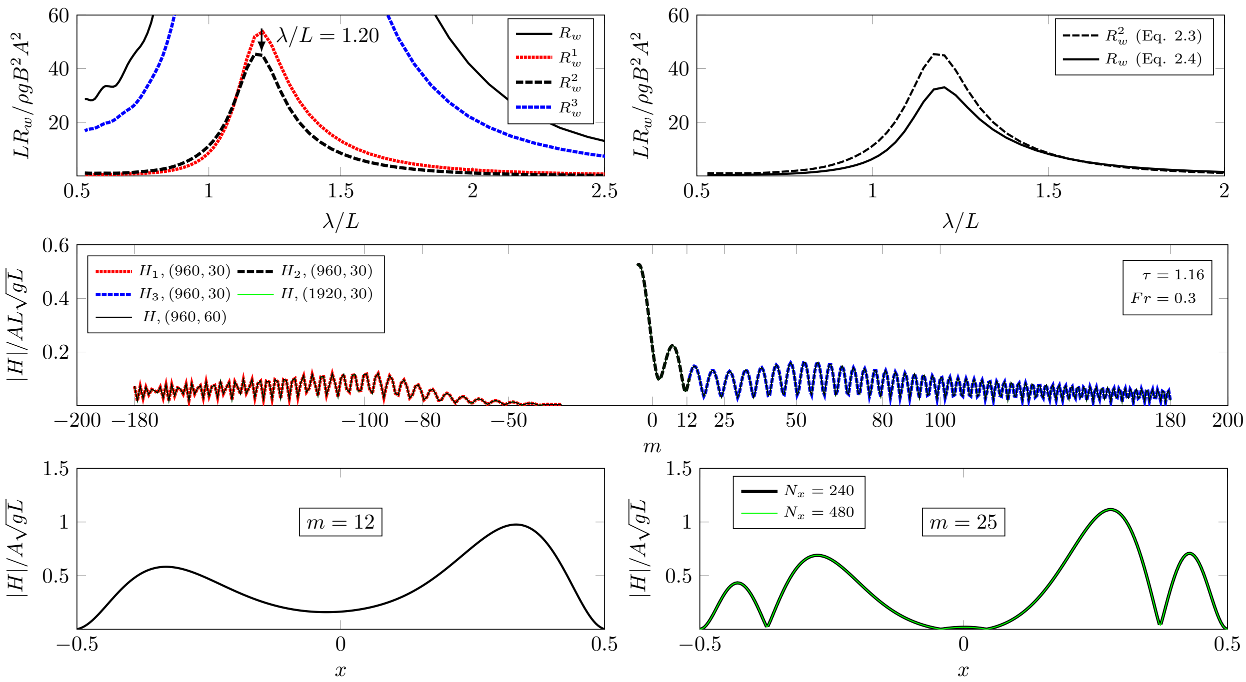

Figure 2: First row: Illustration of relative contribution of the semi-infinite integrals to the added resistance of the Wigley hull based on the ***original*** form of the Kochin function (2.3). Second row: Convergence of the Kochin function in terms of number of 2D sections and number of panels (N_x, N_p). Third row: The Kochin function along the body for $m = 12$ and 25 in case of $\lambda/L = 1.20$.

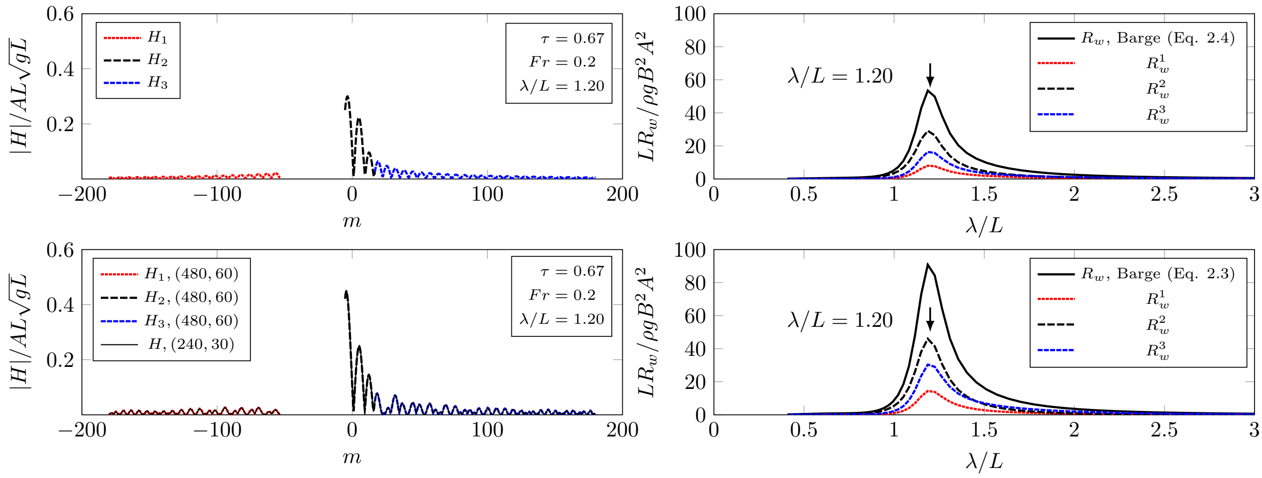

Figure 3: Illustration of relative contribution of the semi-infinite integrals to the added resistance of the barge, $L/B = 10$ and $L/T = 16$ with T as the draft. Top: The computations based on the ***alternative*** form of the Kochin function (2.4). Bottom: The computations based on the ***original*** form of the Kochin function (2.3).

computations are performed using the open-source solver OW3D-Seakeeping which is based on 4th order finite difference-method. Note that these results are based on the Neumann-Kelvin linearization and for $\beta = \pi$

3.2 Floating Bodies (3D)

Two geometries are considered: the Wigley hull type I (free in heave and pitch), and the Modified Wigley hull (free in surge, heave, pitch) as defined by [6]. For both hulls, a convergence study is conducted for the added resistance based on the near-field formulation and Maruo's far-filed method (2.1). See the first and the second rows in figure 4. On the left side, the Kochin functions H_1, H_2, H_3 corresponding to the peak value of the added resistance are plotted. On the right side, convergence for the far-field added resistance (based on only the middle integral) is illustrated. The numbers in the parentheses denote the total number of grid points on the body, normalized by the number of grid points for the coarsest grid. We have not included the contribution from the first and the third integral R_w^1, R_w^3, as this leads to unrealistically large added resistances similar to the 2D computations from the previous section. As the integral for the Kochin function (2.2) contains oscillatory terms, in our computation we ensured that the integral bounds $\bar{k}_2$ and $\bar{k}_3$ are below the grid Nyquist wave number π/l_a. Here l_a is the average grid spacing along the waterline. For the near-field method only the final converged results are shown. Note also that for the Wigley I hull, we have improved our computational accuracy compared with our previous results presented in 32th IWWWFB [7]. As can be seen from the results in figure 4, the converged far-field results (based on only the middle integral) do not compare well with the converged near-field solutions. Moreover it is seen that H_1 and H_3 corresponding to the semi-infinite integrals are also converged.

3.3 Submerged Bodies (3D)

The significance of the semi-infinite integrals is extremely low in the case of submerged bodies. A well-known example from the literature is the added resistance of the fixed submerged spheroid with $L/B = 5$ and the submergence depth of $\hat{d} = 0.75B$. See the results in the third row of figure 4. The BEM solutions are according to [8]. Note that we have included all three integrals from (2.1), even though this was not necessary as the Kochin functions H_1, H_3 are almost zero over the corresponding integration ranges. Apparently this fact that the semi-infinite integrals can be left out of the computation for a submerged body, has been generalized by some researchers to the computations for floating bodies. As we have shown in the previous sections this is not a correct generalization.

4 Conclusions

Using 2D strip-theory and 3D models, we have illustrated the convergence of the calculations for the wave added resistance based on Maruo's method (2.1)-(2.3). By this we have tried to highlight two unsolved problems related to Maruo's formulation and floating bodies. First, the inclusion of the semi-infinite integrals, and second the fact that the added resistance is still predicted to be substantially higher than other reference solutions, even when the semi-infinite integrals are ignored. With regard to the first issue, it is known that some researchers use *poor convergence* as an excuse for introducing an arbitrary submergence of the z- coordinate in the original formulation $e^{\bar{\kappa}(z-\epsilon)}$. By doing so they *damp* these high added resistance values, and can *tune* their models in

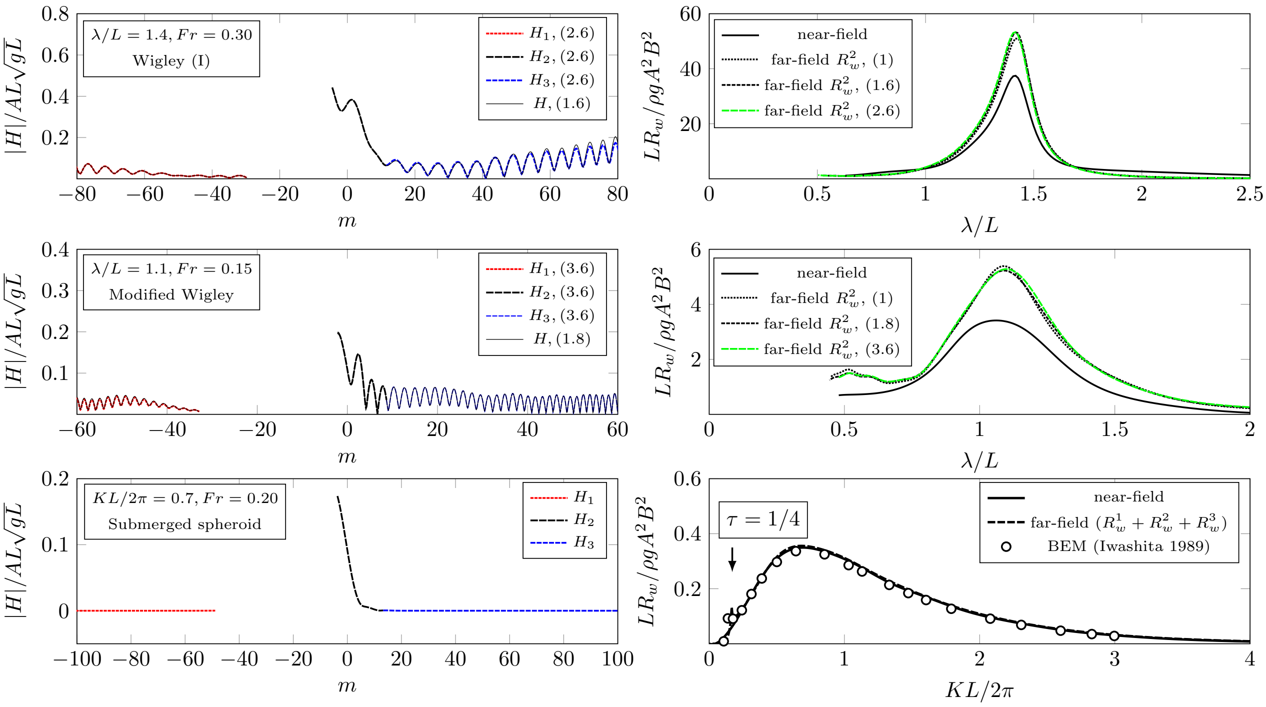

Figure 4: 3D computations based on (2.2). Left: The Kochin function (H_1, H_2, H_3) at the peak value of the added resistance. Right: Added resistance considering only the middle integral R_w^2. for the floating bodies, and considering all three integrals $R_w^1 + R_w^2 + R_w^3$ for the submerged spheroid.

order to achieve good agreement with measurements. With regard to the second problem, [9] has attributed the high added resistances to the fact that potential-flow models overpredict the body motions due to lack of viscous damping. This is in spite of the fact that only the symmetric modes (surge-heave-pitch) are considered in their work. Accordingly they achieve reasonable agreement with experimental data by introducing a damping model to reduce the motion and accordingly the added resistance. Obviously this would have a direct effect also on their near-field solutions, which has not been mentioned in that research. To conclude, we are left with an open question of what is the actual computational problem (if not convergence) in evaluating the integrals and the original forms of the Kochin function (2.2), (2.3) for floating geometries.

Acknowledgment

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References

[1] H. Maruo, “Wave resistance of a ship in regular head seas,” *Bulletin of the Faculty of Engineering, Yokohama National University*, vol. vol. 9, 1960.

[2] M. Kashiwagi, “Prediction of surge and its effect on added resistance by means of the enhanced unified theory,” in *Transactions of the West-Japan Society of Naval Architects 89*, pp. 77–89, The Japan Society of Naval Architects And Ocean Engineers, 1995.

[3] M. Kashiwagi, “Added resistance, wave-induced steady sway force and yaw moment on an advancing ship,” *Ship Technology Research (Schiffstechnik)*, vol. 39, no. 1, pp. 3–16, 1992.

[4] M. Amini-Afshar and H. B. Bingham, “Accurate evaluation of the kochin function for added resistance using a high-order finite difference-based seakeeping code,” in *33th International Workshop on Water Waves and Floating Bodies, Guidel-Plages, France*, 2018.

[5] M. Amini-Afshar and H. B. Bingham, “Added resistance using Salvesen–Tuck–Faltinsen strip theory and the Kochin function,” *Applied Ocean Research*, vol. 106, p. 102481, 2021.

[6] M. Kashiwagi, T. Ikeda, T. Sasakawa, *et al.*, “Effects of forward speed of a ship on added resistance in waves,” *International Journal of Offshore and Polar Engineering*, vol. 20, no. 3, pp. 196–203, 2010.

[7] M. Amini-Afshar and H. B. Bingham, “Implementation of the far-field method for calculation of added resistance using a high order finite-difference approximation on overlapping grids,” in *32th International Workshop on Water Waves and Floating Bodies*, (Dalian, Liaoning, China), 2017.

[8] H. Iwashita and M. Ohkusu, “Hydrodynamic forces on a ship moving with forward speed in waves,” *Journal of the Society of Naval Architects of Japan*, vol. 1989, no. 166, pp. 187–205, 1989.

[9] S. Liu, A. Papanikolaou, and G. Zaraphonitis, “Prediction of added resistance of ships in waves,” *Ocean Engineering*, vol. 38, no. 4, pp. 641–650, 2011.